



## On the Need to Interpret General Relativity

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The main subject of my research is Whitehead's alternative interpretation of the General Theory of Relativity (GTR). This research can only be of value if the business of interpreting GTR is of value. Einstein, Weyl, Eddington, Whitehead, and many others invested a large amount of time searching for the best interpretation of GTR. But pointing at their involvement to justify the importance of interpreting GTR boils down to giving an argument by authority. The invocation of Einstein's glory and fame is insufficient to counter the possible reproach that this kind of research is a waste of precious tax money. Something else and more is needed.

In September 2007 I attended a conference in Budapest, "Mathematics, Physics and Philosophy in the Interpretations of Relativity Theory." I was one of many participants who did not question the need for an interpretation of GTR. I took it for granted. There was, however, a participant who firmly disagreed: Erik Curiel. "What about General Relativity Requires Interpretation?" was the challenging title of his scheduled presentation in defense of the position that GTR does *not* require an interpretation. Its text was included in the conference proceedings (Curiel 2007), and it is by reference to this text, and to my own research, that I will try to respond to Curiel's questioning of the value of the search for a good interpretation of GTR.

### **CURIEL'S CERTAINTY AND GTR'S CLAIMED SELF-SUFFICIENCY**

Erik Curiel opposes the many eminent mathematicians, physicists, and philosophers who hold that general relativity requires an interpretation, and he sketches a defense for the position that there is no need for such a thing—that GTR is self-sufficient in this respect. In the first part of his argument, Curiel proposes a necessary condition for the conclusion

that a scientific theory stands in need of an interpretation, and then proceeds to argue that no problem raised by proponents of the need for an interpretation satisfies the condition. Curiel is aware that his argument has only so much force as the necessary condition he proposes, and that he cannot discuss all relevant problems. His discussion is restricted to problems implied by the Hole Argument, by the invariance of GTR under the group of diffeomorphisms, and by the search for a theory of quantum gravity. In the second part, Curiel compares the situation for GTR with that for quantum mechanics. He links the requirement of an interpretation for quantum mechanics with its measurement problem and then argues that GTR, given its lack of a similar problem, does not require an interpretation. Q.E.D. Or not? Let's look into some details of Curiel's paper.

*What sort of interpretation is required, if any?*

According to Curiel, a physical theory is a framework that provides the resources to construct appropriate models of physical systems, and the required understanding of a physical theory lies in the fact that we can use its models to represent experiments, to plan and perform those experiments, to comprehend their results, and to produce novel predictions. Consequently, Curiel's necessary condition for a physical theory to stand in need of an interpretation is that we cannot achieve the required understanding of that theory "without recourse to the resources of an extra-theoretical framework." In other words, only if a physical theory fails to supply us with the appropriate models to understand our experimental results, and if we have to take recourse to an additional framework providing other and more appropriate models, then we can say that a theory requires an interpretation. Curiel writes:

Let us say that we have a [theoretical] framework some aspect of the semantic content of which is, for one reason or another, poorly comprehended. One natural route of attack in attempting to comprehend it better is to try to find another [extra-theoretical] framework that we do comprehend better, in terms of which we can construct a representation of the poorly comprehended part of the first. The representation must in some important sense recapitulate in the second framework the poorly comprehended structure of the first—it must preserve the structure. This idea, the "preservation of structure", plays the crucial role. Klein's model of Lobachevskian geometry in the Euclidean plane provides a wonderful example. (32)

Curiel holds—and I could not agree more—that physics is all about modeling, and that, somehow, appropriate modeling involves common relational structures. However, he limits the required understanding of the physicist to being able to model experiments. In other words, Curiel limits his condition for the necessity of an interpretation to the experimental context. He does not talk about the need for an interpretation of the non-observational terms and concepts of a physical theory, as far as they are not invoked in the modeling of experiments, let alone about the need to go beyond the context of justification in order to understand a theory in its context of discovery. At this point, I only mention this limitation and postpone my Whiteheadian critique of it.

Even while provisionally accepting Curiel's limitation of the semantic dimension of any framework to its experimental meaning, I cannot resist highlighting the fact that Curiel's wonderful example—Klein's model of Lobachevskian geometry—seems to point in the opposite direction of what he sets out to show. Indeed, if we do not only stress the structural similarity between an appropriate Euclidean model and the non-Euclidean geometry it models, but also stress the necessity of such a model to be able to understand such a geometry, then the example seems to imply that GTR, given its pseudo-Riemannian spacetime geometry, *does* require an interpretation. Of course, this issue is a non-issue if we decide that the Euclidean models of the non-Euclidean geometry of GTR are part of its own resources. Unfortunately, however, then another issue arises. How to distinguish between theoretical and extra-theoretical modeling resources in order to evaluate the need of a physical theory for an interpretation? Instead of delving into either of these two issues now, I turn to the first problem Curiel's paper deals with, the problem of whether or not Einstein's Hole Argument implies that Curiel's condition for the necessity of an interpretation of GTR is fulfilled.

### *The Hole Argument*

Curiel writes:

In brief, the Hole Argument claims that a distribution of ponderable matter sources would not by itself serve to fix the physical state (the “gravitational field”) of a 4-space that itself contains no ponderable matter but is bounded by that distribution. Let  $O$  be an open region in spacetime devoid of ponderable matter but entirely bounded by it. . . . Einstein saw it, initially, as a flaw in a covariant field theory that a distribution of ponderable matter sources would not by itself

serve to fix the explicit form of the metric tensor in a region such as  $O$ , but rather would fix it up to diffeomorphism. He took this to show that such theories manifest an unacceptable form of indetermination. Consensus today seems to have it that the proper way of understanding the Hole Argument places it rather at the heart of the question about the ontic status of spacetime points. Its lesson, so claimed, is that one cannot identifiably individuate spacetime points without reliance on metrical structure, that there is no “bare manifold of points” under the metrical field. (33-34)

In other words, Einstein’s initial Hole Argument against general covariance involved the fact that in a generally covariant GTR the pre-existing points of particular regions of spacetime—the so-called “holes,” containing no ponderable matter—can be attributed different gravitational fields (different spacetime metrics) for the same overall source distribution of ponderable matter:

The problem with generally covariant field equations, according to the hole argument, is that they allow one and the same source to produce what look like different metric fields, whereas the job of field equations is to determine uniquely what field is produced by a given source. (Janssen 15)

Einstein justified his early proposal of a not generally covariant GTR in 1913 by stressing that each generally covariant alternative would fail to determine a unique gravitational field (a unique spacetime metric) from a given source distribution of ponderable matter. However, Einstein dropped his resistance to general covariance by the time that he formulated his ultimate proposal of a general covariant GTR in 1915. The consensus today holds that Einstein ultimately grasped that there are no pre-existing points of spacetime, that spacetime cannot exist apart from the gravitational field (apart from its metric):

The escape from the hole argument is that on closer examination the different fields compatible with the same source turn out to be identical. The hole argument rests on the assumption that spacetime points can be individuated and identified before any of their spatio-temporal properties are specified. Reject this assumption and the argument loses its force. The allegedly different metric fields only differ in that different featureless points take on the identity of the same space-time points. If space-time points cannot be individuated and identified independently of their spatio-temporal properties, this is no difference at all. (Janssen 16)

Einstein's refusal to accept the existence of a topological spacetime structure apart from any spacetime metric is said to have eliminated the Hole Argument against the current formulation of GTR. But this consensus implies that an adequate understanding of (the general covariance of) GTR requires an ontological interpretation. In other words, this consensus implies that Curiel's condition for the necessity of an extra-theoretical interpretation of GTR is met. Curiel, however, questions the importance of the consensus view on why Einstein dropped the Hole Argument against general covariance, and rejects the implication that in order to fully understand GTR, we have to take recourse to an ontological framework. Curiel claims to understand the generally covariant character of GTR without stipulating the ontic status of the spacetime concepts of point, topology, metric, etc. He writes:

Nothing I can see militates in favor of taking the Hole Argument as exhibiting something of intrinsic physical significance that our current understanding of general relativity cannot comprehend. . . . It is no more a problem that the distribution of ponderable matter by itself does not fix the explicit diffeomorphic presentation of the metric than it is that the distribution of charges in Maxwell theory does not fix a unique solution of the Maxwell equations. Indeed, I should think it much less of a problem, for in the case of the Maxwell field we cannot determine a *physically* unique solution without imposing boundary conditions: Otherwise, we are always free to add a field with vanishing divergence and curl to a solution to yield another that will have different physical effects on charged bodies. In general relativity, one does not need to do anything of the sort to determine a physically unique solution. . . . All observers, no matter which presentation of the metric they use in their respective models, will agree on the physically relevant parts of the outcome of all experiments. (34)

Curiel defines the intrinsic physics of a theory as “what all possible observers in all possible states as schematically represented in a model of an experiment would agree on, no matter the presentation of the theory used to produce the models.” Given the fact that the non-uniqueness of the gravitational field solutions of Einstein's covariant field equations does not imply any indeterminism in the experimental context, or at least no more than the non-uniqueness of the electromagnetic field solutions of Maxwell's equations, Curiel infers that we do not need any ontology to clarify matters of intrinsic physical significance. In other words, Curiel rejects the consensus

view that GTR needs an ontological interpretation to minimize the non-uniqueness impact of general covariance, simply because this impact is already minimal or even zero in the context of experiments.

After concluding that an adequate understanding of the intrinsic physics of GTR does not require an ontological interpretation of the notions of spacetime and field, Curiel adds:

To see the Hole Argument as a problem in general relativity that requires for its resolution an interpretation of the theory, one must believe ponderable matter to be “privileged” in some deep sense over metrical structure. . . . Crudely put, I think that those who hold the view that the Hole Argument leads to indeterminism or bears on the ontic status of spacetime points assume that the equation expresses some sort of causal statement of the form, “ponderable matter gives rise to, determines, spacetime metrical structure.” . . . I think a more helpful view of the [Einstein field] equation can be had by looking at the analogous question of the relation between force and acceleration as expressed in Newton’s Second Law. Applied force, in Newtonian mechanics, does not “cause” or “give rise to” acceleration in any deep sense. One may as well regard acceleration as just the measure of the applied force as one may regard the applied force as just the measure of acceleration. (35-36)

Einstein’s field equation relates the metric tensor (expressing spacetime manifold and gravitational field as one) to the energy tensor (incorporating the distribution of ponderable matter, electromagnetic field energy, momentum, stress, etc.). According to Curiel, the Hole Argument only implies that GTR is in need of an ontological interpretation if the Einstein field equation is taken to express a causal relation running from ponderable matter to spacetime structure, from energy tensor to metric tensor. But according to Curiel, we can conceive the fundamental relation of GTR as a non-causal relationship, similar to the bi-directional relationship between force and acceleration in Newton’s Second Law. It does not only run from matter to metric but also in the opposite direction, and so it is equally true “to say that metrical structure in general relativity determines the distribution of ponderable stress-energy” (37). The comparison with Maxwell’s equations again holds. Maxwell’s equations also express that the electromagnetic field can determine the distribution of charged matter.

Whitehead, as well as the older Einstein, rejected Einstein’s initial Machian interpretation of the field equation of GTR—an interpretation in which, indeed, ponderable matter causally determines the spacetime

structure. Nonetheless, I doubt that interpreting Einstein's equation as expressing a kind of mutual harmony between matter and field, rather than a causal influence from matter to field, is sufficient to dismiss any discussion of their ontological status. Maybe this harmony requires a Whiteheadian interpretation, that is, an ontological interpretation in which both material particles, and physical fields, are abstractions from one overall process? But even if I take Curiel's epistemological excursion as successfully dismissing this ontological issue, yet another problem remains unresolved. If Curiel invokes a particular epistemological interpretation of Einstein's field equation to strengthen his claim that GTR is not in need of ontological interpretation, then he tacitly illustrates the view of his opponents, that some kind of extra-theoretical interpretation is needed anyway.

#### *Diffeomorphic freedom*

The next problem Curiel discusses is "the invariance of solutions to the Einstein field equations under the action of diffeomorphisms," or better, the fact that this invariance "embodies an irremediable mathematical ambiguity in the apparatus provided by general relativity for the modeling of experiments." According to Curiel, however, the fact that "the choice of the presentation of spacetime manifold and metric one uses to model an experiment is fixed only up to diffeomorphism" does not require recourse to any extra-theoretical interpretation. He writes:

A comparison is edifying. Classical mechanics, as embodied in Lagrangian and Hamiltonian mechanics, shares similar ambiguities, slightly different in each formulation of the theory. . . . More to the point, it is clear in these cases that the physical significance of the theory's models is not masked or polluted by the unavoidable arbitrariness in the details of their presentations. In the same way, the diffeomorphic freedom in the presentation of relativistic spacetimes does not *ipso facto* demand an interpretation, in so far as it in no way prevents us from focusing on and investigating what is of true physical relevance. (38)

Again, Curiel limits physical significance and relevance to the experimental context, and this presupposition invalidates the conclusion that diffeomorphic freedom in GTR requires the use of extra-theoretical machinery to grasp what is of true physical significance and relevance in GTR. Before GTR was established, Curiel writes:

It was not clear what it could have meant for the representation of physical systems to be invariant under the full group of spacetime diffeomorphisms, as Einstein's struggle with the Hole Argument poignantly shows. The establishment and comprehension of general relativity itself provides an interpretation for the significance of diffeomorphic invariance, a significance we do understand well in the context of the theory: To transform a model of a physical system by the action of a spacetime diffeomorphism does nothing more than change the presentation of the model, but does not alter the intrinsic physics that the model depicts. It is an inevitable ambiguity in our mode of presentation in the theory. (40)

### *Quantum gravity*

Curiel then deals with the problem that the search for a theory unifying GTR and quantum mechanics seems to require a proper interpretation of GTR. He introduces the problem as follows: "Many eminent workers in the field of quantum gravity suggest that at least part of the reason we have so far had little if any success in the search for a viable theory points to some lack of understanding of the conceptual resources of general relativity. We require a proper interpretation of general relativity, they argue, in order to find a path to the deeper theory" (42). Curiel's strategy to counter this argument is twofold.

Firstly, Curiel argues that it is not plausible to hold that the resolution of conceptual difficulties in GTR, if there are any, would provide clues to a theory of quantum gravity. If GTR needs to be replaced by a more fundamental theory, it is not itself such a theory: "Those who hold that general relativity requires an interpretation in order that it may suggest paths to a theory of quantum gravity *a fortiori* do not view general relativity as a 'fundamental' theory, whatever exactly that may come to" (41). And if GTR is not a fundamental theory, it is difficult to see why an interpretation of GTR that clarifies its concepts (spacetime metric, gravitational field, etc.) should give us an understanding of the structure of the physical world in such a way as to illuminate a path to a viable theory of quantum gravity: "It is difficult to see . . . how an interpretation of any higher-order physical theory could shed light on the conceptual resources of a more fundamental theory to which it reduces" (41).

Secondly, Curiel argues that it is not plausible to hold that the difficulties that the standard procedures of quantization face when applied

to GTR can be attributed to a failure on our part in understanding GTR. Curiel points out

that we ought not be “quantizing” anything in the first place. Maxwell and Boltzmann did not “statisticalize” Navier-Stokes theory in order to arrive at a viable account of molecular kinetics. . . . They based their investigations to a large part on the known, well-entrenched body of experimental knowledge they were trying to produce a better theory of. . . . That we find ourselves in the position of having to attempt to work from the top down in the case of quantum gravity, trying to base our search on the conceptual resources of theories thought to be less fundamental, rather than tackling the formulation of such a theory heads on, shows that we lack experimental knowledge, not necessarily conceptual understanding, at least with respect to the resources of general relativity—we need hard experimental data that we do not know how to accommodate in the context of general relativity to move forward in the understanding of the physical regime that general relativity treats. (43-44)

When Curiel turns to the search for quantum gravity, he turns his attention away from the context of justification, and towards the context of discovery. But even in this context, he rejects the need for an interpretation of GTR, because he considers confrontation with adverse experimental data, rather than conceptual speculation, to be the motor of discovery.<sup>1</sup> However, if we compare the search for quantum gravity with Einstein’s own search for relativistic gravity, we can easily arrive at just the opposite view. True, in 1913 Einstein characterized his search for a relativistic theory of gravitation as “a hopeless undertaking” because of the lack of hard experimental data to guide him, and he gave the following comparison:

To see this clearly, one need only imagine being in the following analogous situation: Suppose that of all electromagnetic phenomena, only those of electrostatics are known experimentally. . . . Who would have been able to develop Maxwell’s theory of electromagnetic processes on the basis of these data? (“Present” 544)

But Einstein *did* arrive at GTR after a decade long and difficult speculative adventure; he *was* mainly guided by conceptual principles that stretch to the point of breaking the ones underlying the special theory of relativity (STR).

Anyway, Curiel writes: “In the absence of empirical data that show us where general relativity goes wrong—for we have none—it makes sense to focus on the theory that we manifestly do not understand” (45).<sup>2</sup>

This sentence has a dual meaning. It does not only mean that, contrary to the clarification of the conceptual resources of GTR, the clarification of the conceptual resources of quantum mechanics might provide clues to a theory of quantum gravity. It also implies that, contrary to GTR, quantum mechanics is in need of such a clarification, and hence, of an interpretation. Let us now see why Curiel thinks so.

### *Comparison to Quantum Mechanics*

According to Curiel, quantum mechanics is in need of an interpretation because of its measurement problem, and he writes:

With regard to the impetus for demanding an interpretation of a theory, the difference between general relativity on the one hand and quantum mechanics on the other could not be more complete: It is the difference between ambiguity in the machinery of the former theory that models experiments, and ambiguity as to what counts, according to the latter theory, *as* an experiment. . . . Quantum mechanics demands an interpretation because it is not clear how to model physical phenomena, how to model the outcome of experiments . . . There is no analogous problem in general relativity. We know how to model in the terms of the theory experiments that manifest and probe every phenomena suggested or predicted by the theory, with no inconsistency of any kind, for we understand with no lack of cogency the fundamental, physical terms and principles of the theory in which one articulates its models and draws conclusions on their basis. In quantum mechanics, we do not even know what the fundamental terms and principles are—“measurement”? “interaction”? “observation”? (44-45)

The need to interpret a theory is linked by Curiel to the failure of that theory's conceptual resources to model experiments. Thus, Curiel judges quantum mechanics as in need of an interpretation. Indeed, the measurement problem of quantum mechanics *is* the problem of not being able to explain quantum experiments in quantum mechanical terms; it *is* the problem of not being able to model the interaction of quantum events with macroscopic devices, without taking recourse to extra-quantum mechanical resources. However, Curiel's certainty on the non-existence of a similar measurement problem in GTR might be shaken by some of Einstein's own remarks. For instance:

The idea of the measuring-rod and the idea of the clock . . . do not in the conceptual edifice of physics play the part of irreducible elements, but that of composite structures, which may not play any

independent part in theoretical physics. But it is my conviction that in the present stage of development of theoretical physics these ideas must still be employed as independent ideas; for we are still far from possessing such certain knowledge of theoretical principles as to be able to give exact theoretical constructions of solid bodies and clocks. (“Geometry” 35-36)

Clearly, Einstein was struck by the fact that GTR

introduces two kinds of physical things, i.e., (1) measuring rods and clocks, (2) all other things, e.g., the electro-magnetic field, the material point, etc. This, in a certain sense, is inconsistent; strictly speaking measuring rods and clocks would have to be represented as solutions of the basic equations (objects consisting of moving atomic configurations), not, as it were, as theoretically self-sufficient entities. However, . . . it was better to permit such inconsistency—with the obligation . . . of eliminating it at a later stage of the theory. (“Autobiographical” 59-60)

Clearly, Curiel has not arrived at the stage of eliminating this inconsistency with regard to (at least some of) the measurement devices of GTR. Hence, how can he be so convinced that, unlike quantum mechanics, GTR is theoretically self-sufficient? How can he be so certain that, contrary to the measurement problem in quantum mechanics, there is no issue in GTR to model experiments, and hence no need for any extra-theoretical interpretation?

#### **THE WHITEHEADIAN ANTIDOTE TO CERTAINTY AND SELF-SUFFICIENCY**

Physics is all about modeling, and successful modeling seems to imply the fact that somehow an appropriate model has a relational structure in common with what it models. This is Curiel’s point of departure, and I take the same point of departure to discuss the Whiteheadian view on issues raised by Curiel’s paper. There is no divergence between this point of departure and Whitehead’s view on physics, apart from the fact that Whitehead does not speak about “models” (as philosophers of science tend to do since the rise of model theory), but about “analogies,” and sometimes about “metaphors.” Also, in most cases, he uses the more general expression “pattern” instead of “relational structure,” hence including not only structural, but also qualitative aspects.

So for Whitehead, physics is all about analogies, about the recognition of similarity amid diversity. Each analogy reveals a pattern that the two subjects of the analogy—the clarifying and the clarified, the source and the

target—have in common. Of course, each analogy only reveals a partial similarity of pattern, and hence, is characterized by a tension between similarity and diversity. Nonetheless, for Whitehead, the discovery of fertile analogies or metaphors is the motor of the progress and success of rational thought in general, and physics in particular. He writes:

The procedure of rationalism is the discussion of analogy. The limitation of rationalism is the inescapable diversity. The development of civilized thought can be described as the discovery of identities amid diversity. (*MT* 98)

The limitation of rationalism immediately puts an end to all claims of certainty! If there were exact analogies, then there would be certainty, but such an adjective does not apply to our knowledge, which involves “a baffling mixture of certainty, ignorance, and probability” (*PR* 205). According to Whitehead, “rationalism is an adventure in the clarification of thought,” it is “progressive,” but “never final,” it is “an adventure in which even partial success has importance” (*PR* 9).

Despite their common starting point, Curiel and Whitehead diverge. Whereas Curiel attempts to draw a line between theoretical models and extra-theoretical models, only calling the latter models interpretations, there is no such demarcation in Whitehead. For Whitehead, each analogy is an interpretation (a partial interpretation). Of course, the evolution of physics has manifested a characteristic preference for a particular type of analogy—that is, for analogies that reveal mathematical patterns. In this sense, a distinction between physics and other disciplines of thought can be made, but it should not be made too strict:

No one would study geology as a preparation for appreciation of the sonnets of Shakespeare or the fugues of Bach. The things discussed in geology are so different from sonnets and so different from fugues. The result is that the interconnections discussed in a treatise on geology are very different from those disclosed in the structure of a sonnet or of a fugue. But faint analogies do occur. Sometimes these analogies rise in importance. For example, the Greeks discovered analogies between the lengths of strings and the harmonies of musical notes, and between the measurements of the dimensions of a building and the beauty of the structure. (*MT* 98)

Also, whereas Curiel restricts the semantic dimension of a conceptual framework to its experimental meaning, there is no such limitation in Whitehead. For Whitehead the semantic value of a conceptual framework is its capacity to add and acquire meaning, not only in the context of

justification, but also in the context of discovery; not only in the context of experimentation, but in whatever context of experience. Of course, the evolution of physics has manifested a characteristic preference for a particular type of experience—that is, for experimental experience. Experiments are indeed the ultimate arbiters in the game of physics, but they should not be the only ones. The experimental context is indeed its main playground, but it should not be the only one. Even in the context of justification, according to Whitehead, “there are two gauges through which every theory must pass.” Of course, there is the experimental gauge, “the narrow gauge which is . . . the habitual working gauge of science.” But there is also an experiential gauge through which a theory must pass, “the broad gauge which tests its consonance with the general character of our direct experience” (*R 3-4*).

For Whitehead, physics’ preference for mathematical patterns and experimental experiences should not lead us to ignore the analogical nature of its explorations, nor the requirement of coherence of its conceptual results with its experiential background. The unavoidable experiential background dependence of each theory, never to be completely analyzed, immediately puts an end to all claims of GTR’s self-sufficiency! And Whitehead adds:

The notion of the complete self-sufficiency of any item of finite knowledge is the fundamental error of dogmatism. . . . Wherever there is the sense of self-sufficient completion, there is the germ of vicious dogmatism. There is no entity which enjoys an isolated self-sufficiency of existence. (*ESP 101-02*)

Whitehead embraced “the doctrine of Pattern” (*ESP 102*), emphasizing that mathematics is all about patterns, and physics is all about the partial disclosure of a largely unanalyzed universe by means of pattern recognition. Through the glasses of contemporary philosophy of science, Whitehead’s doctrine of patterns can be seen as a structuralist philosophy of mathematics (see Shapiro, *Thinking 257-89*; for more detail, Shapiro, *Philosophy*), and as a structural realist philosophy of physics (see Worrall as well as Daston and Galison 253-307). I will now further clarify Whitehead’s “structuralist” view, and relate it to each of the issues discussed by Curiel.

### *Mathematical patterns*

It is of utmost importance to see that even within mathematics there are different levels of abstraction, different levels of purity. A less abstract

piece of mathematics can be used to embody a more abstract piece of mathematics. A popular example is the geometrical representation of the complex numbers. Consequently, a less abstract piece of mathematics can function as the clarifying subject in an analogy with a physical subject in need of clarification, and with a more abstract mathematical pattern as the similarity pattern that links source and target subject. An example is provided by the well-known parallelogram rule for the addition of forces in physics. The diagonal of a parallelogram can function in an analogy to clarify a compound force, but the addition of vectors is the more abstract mathematical pattern the geometrical subject has in common with the physical subject.

Whitehead was both a pure and an applied mathematician, and these two dimensions of his mathematical career were closely linked. He saw the development of pure mathematics as a condition for the development of applied mathematics. To be more specific, Whitehead saw the development of pure mathematics as the development of a repository of relational structures that are, or eventually could be, applied in physics and philosophy. Moreover, for him, a relational structure is applied in physics when it functions as the pattern of similarity of one of the analogies constituting physics.

Whitehead's dual interest in mathematics was inspired by two major historical events. The first was Maxwell's achievement in the domain of electricity and magnetism. Maxwell, inspired by Faraday and Kelvin, clarified electric and magnetic phenomena by a succession of analogies—the analogy with fluid mechanics, the analogy with ether mechanics, and the analogy with Lagrangian and Hamiltonian dynamics. In his 1873 *Treatise on Electricity and Magnetism*, Maxwell formulated the mathematical patterns of similarity thus discovered in terms of the scalar and vector parts of Hamilton's quaternions, and also by means of Green and Stokes' integral theorems. Consequently, it is Maxwell and his followers (Heaviside, Hertz, etc.) who initiated understanding electricity and magnetism in terms of the interplay between scalar and vector fields, that is, in terms of the operations of divergence, curl, and gradient, connecting scalar distributions of charge, scalar and vector potentials, electric vector fields, and magnetic vector fields.

In 1884, only eleven years after its publication, Whitehead chose Maxwell's *Treatise* as the subject of his Trinity College fellowship dissertation at Cambridge. Maxwell's achievement (as well as that of some

of his Cambridge followers—Poynting, J.J. Thomson, and Larmor) made an everlasting impression on Whitehead. However, in order to understand Whitehead's view on mathematics, a second historical event needs to be taken into account. It is the emergence of pure mathematics, a development in which Whitehead played a crucial role. Whitehead was inspired by the evolution of mathematics into a realm of abstraction way beyond the realm of numbers and figures. Of major importance for him in this context were Grassmann's calculus of extensions and Russell's logic of relations.

Grassmann's theory was the starting point of Whitehead's decade-long research of algebraic patterns and their applicability in physics, resulting in his 1898 *Universal Algebra with Applications*. Russell's theory was the starting point of Whitehead's decade-long collaboration with Russell, resulting in the publication of the 1910, 1912, and 1913 volumes of *Principia Mathematica*.

In *UA*, mathematics became abstract beyond numbers and coordinates, and beyond 3-dimensionality and Euclidean geometry, but its abstraction did not completely overcome its geometrical origins. Disregarding the part on algebraic logic, and the attempt to distinguish a pure algebraic notion of space from the applied geometrical notion of physical space, the title of *UA* might just as well have been: *Geometric Algebra for Physicists*. Nonetheless, the appreciation for this Whiteheadian approach is still growing today, as is clear from a recent and major publication under exactly this title—*Geometric Algebra for Physicists* (Doran and Lasenby). So even in retrospect, we can say that Whitehead's election, because of *UA*, as a member of the Royal Society of London was fully justified.

However, with *PM*, a decisive next step of abstraction towards a truly pure mathematics was taken, including emancipation from its Greek numerical and geometrical origins. Instead of attempting to capture Boole's logic and Grassmann's calculus in the mathematical framework of a universal algebra, the aim changed to capturing all of mathematics in the framework of a pure mathematical logic. The pure mathematico-logical patterning of *PM* reveals the ambition of its authors to form a unifying substructure for the whole body of mathematics, including all its arithmetical, algebraic, and geometrical limbs. But at the same time, it is an outgrowth of Whitehead's original ambition to construct a unified repository of purely abstract relational patterns for application in physics.

To make perfectly clear the relevance of what has been said, consider another, very simple analogy, keeping in mind that an analogy is all

about the recognition of a common pattern in different situations. Suppose I say: “that Coke bottle stands on my desk as if it were a straight line perpendicular on a plane,” then I articulate a geometrical analogy. The semi-intuitive, semi-abstract, Euclidean geometry of straight lines, planes, perpendicularity, etc., functions as the clarifying subject of the analogy (it clarifies how the bottle stands on the desk). But next to that, a pure geometry of logical relations, defining the relational structure of perpendicularity in a completely abstract way, functions as the supplier of the pattern of similarity of the analogy. This reminds me of the famous remark ascribed to Hilbert, that in pure geometry one should always be able to say, instead of “points, lines, and planes,” “tables, chairs, and beer mugs” (see Gray 179-80). Pure geometry involves the construction of relational structures of pure *its*; it involves the relational essence of situations, defined in terms of pure logical subjects (variables) without individual essence. Pure geometry is only so called because of the possibility of its semi-intuitive, semi-abstract geometrical embodiment. So in what follows, I will only use the term “geometry” to denote “semi-intuitive, semi-abstract geometries,” and I will always use the term “pure mathematics” when the construction of purely abstract relational structures is at stake.

Paul Valéry writes: “Mathematics is the science of acts without things—and through this, of things one can define by acts” (811).<sup>3</sup> With some generalization, I would paraphrase Valéry’s aphorism by saying that pure mathematics is the science of relations without *relata*—and through this, of *relata* one can define by relations. In pure mathematics, according to Whitehead, “we are entirely abstracting from any consideration of any particular entities, or even of any particular sorts of entities. . . . We are merely thinking of those relationships . . . which are entirely independent of the individual essences” (*SMW* 19-20). And he adds:

This is a very remarkable feat of abstraction; and it must have taken ages for the human race to rise to it. During a long period, groups of fish will have been compared to each other in respect to their multiplicity, and groups of days to each other. But the first man who noticed the analogy between a group of seven fishes and a group of seven days made a notable advance in the history of thought. He was the first man who entertained a concept belonging to the science of pure mathematics. (*SMW* 20)

Whitehead presents the elementary and familiar concept of number by means of the notion of analogy. Each number is a pattern of similarity

of diverse situations, taken together in analogy. This Whiteheadian view on the value of analogical thinking and on the nature of numbers has been clearly understood and articulated by Suzanne Langer, one of Whitehead's famous Harvard students. Langer presents the view in terms of "forms" instead of "patterns," "contents" or "integuments" instead of "embodiments," and "logical" instead of "pure mathematical":

The great value of analogy is that by it, and it alone, we are led to seeing a single "logical form" in things which may be entirely discrepant as to content. The power of recognizing similar forms in widely various exemplifications, i.e. the power of discovering analogies, is logical intuition. . . . The consideration of a form, which several analogical things may have in common, apart from any contents, or "concrete integuments," is called *abstraction*. . . . If we note what is common to a couple of days, a pair of gloves, a brace of partridges, and a set of twins, we are abstracting a form which each of these items exhibits, namely its numerosity, *two*. If we speak simply of 'two-ness' or 'two,' we are treating of this form *in abstracto*. Or again, if we consider the order in which hours of a day follow each other . . . and then regard . . . the succession of volumes of the *Encyclopaedia Britannica*, or the sequence of Presidents of the United States, we see at once that there is a common form in all these progressions. They are all analogous, all different contents for a pattern which is a section of the *ordinal number series*: first, second, third, etc. It is easy to see that it is but a short step from the recognition of analogies, or different contents for the same form, to abstraction, or the apprehension of that form regardless of any particular content. (33-34)

Even though it is easy to see that cardinal numbers (one, two, three, . . .) and ordinal numbers (first, second, third, . . .) are common features of diverse situations, it is a very complex task to express them as purely abstract patterns (as logical forms). When we think of patterns, we spontaneously think of geometrical patterns, and of course, as Whitehead writes, we might recur to "the Greek mode of representing numbers by patterns of dots," for then "the notions of number and of geometrical configuration are less separated than with us" (*SMW* 28). However, we should not make the mistake of confusing the purely abstract patterns Whitehead and Russell aimed at by means of the logic of relations with the semi-intuitive, semi-abstract, geometrical patterns, figures, or shapes, which were central to Greek mathematics. "The function in thought which is performed by pure mathematics," Whitehead reminds us, "is a

resolute attempt . . . to separate the elements of mere matter of fact from the purely abstract conditions which they exemplify" (*SMW* 24). And he warns us not to misunderstand this resolute attempt to separate the various particular embodiments from the purely abstract patterns they have in common:

This fact, that the general conditions [= the purely abstract patterns] transcend any one set of particular entities, is the ground for the entry into mathematics, and into mathematical logic, of the notion of the "variable." It is by the employment of this notion that general conditions [= purely abstract patterns] are investigated without any specification of particular entities. This irrelevance of the particular entities has not been generally understood: For example, the shape-iness of shapes, e.g. circularity and sphericity and cubicality as in actual experience, do not enter into the [pure] geometrical reasoning [= the pure mathematical reasoning]. (*SMW* 25)

When we use the terms "form" and "matter," and stress that analogical thinking reveals similarity of form amid diversity of matter, we "only reinforce our natural prejudice in favor of imagining form as *shape*, and whatever has form, as *stuff*" (Langer 26). Whitehead warns us of exactly this prejudice when he says we should not think that pure mathematics is about the shape-iness of shapes.<sup>4</sup> When a pure mathematician, e.g., deals with a sphere, he is not focused on the shape of a soccer ball, but constructs purely abstract relations between purely abstract logical subjects or variable elements. A purely mathematical sphere is arrived at by first fixing an element  $c$  in any set of elements  $V$  on which a distance function is defined, and then constructing the subset  $S$  of  $V$ , consisting of all elements  $x$  at a fixed distance  $r$  of  $c$ . In pure mathematics the focus is on the relational essence that all particular spheres have in common, and not on their shape-iness, which belongs to their particularity. In fact, by appropriately changing the distance relation in the purely abstract definition of a sphere, the particular shape of the geometrical embodiment of the purely mathematical sphere can be changed, e.g., from spherical to cubical.<sup>5</sup>

Again, Suzanne Langer clearly understood and articulated this Whiteheadian difference between the relational essence and the shape-iness of shapes when she wrote:

The meaning of "form" is stretched beyond its common connotation of shape. In the science or art of musical composition we speak of a rondo form, sonata form, hymn form, and no one thinks of material

shape. Musical form is not material; it is *orderliness*, but not *shape*. So we must recognize a wider sense of the word than the geometric sense of physical shape. . . . In this wider sense, anything may be said to have form that follows a pattern of any sort, exhibits order, internal connection, . . . The bridge that connects all the various meanings of form—from geometrical form to the form of ritual or etiquette—is the notion of *structure*. (23-24)

The preceding discussion might seem irrelevant. Why focus on the difference between pure mathematics and its Greek origin as a geometry of number and shape? Why spend time to highlight the difference between form (or pattern, or structure) and geometrical shape? However, this discussion is of major importance to highlight Whitehead's divergence from Einstein. According to Whitehead, Einstein did not overcome our natural prejudice to consider form as the shape of stuff, and did not go beyond the ancient fixation on "the shape-iness of shape, which is an impure mathematical entity." On the contrary, Whitehead adds, "when Einstein and his followers proclaim that physical facts, such as gravitation, are to be constructed as exhibitions of peculiarities of spatio-temporal properties, they are following the [ancient] tradition" (*SMW* 28). In other words, when Einstein and his followers proclaim that the pattern underlying gravitational phenomena is to be identified with the variably curved shape of a kind of spatio-temporal substance, they stick to the natural and ancient interpretation of a purely abstract relational structure in terms of geometry, whereas we might discover a more appropriate interpretation of the same pure mathematical pattern, if only we were able to shake off our property-substance thinking in general, and our shape-stuff prejudice in particular.

#### *Interpretation by analogy*

So let us return to GTR, and to Curriel's claim that GTR is not in need of an interpretation. In GTR, Einstein and his followers—e.g., Weyl and Eddington—describe the gravitational field as a pseudo-Riemannian spacetime, and the gravitational paths of light and free falling particles as geodesics—as straightest lines in the variably curved pseudo-Riemannian spacetime. Given what I said above, this only means that Einstein and others, in order to understand gravitational phenomena, take recourse to a geometrical model, even though it is a non-Euclidean model. Moreover, the force of this model (the strength of this Einsteinian analogy) lies in the relational structure that these particular phenomena and this particular

geometry have in common. Consequently, and strictly speaking, this relational structure (or pattern) is the core of GTR.

However, if we take this core of relatedness out of the modeling context (out of the context of analogical thinking), it is just a piece of pure, non-applied mathematics. Because GTR *is* applied mathematics, we are encouraged to claim that GTR acquires meaning because its core is part of a model (is the pattern of an analogy)—*in casu*, the pseudo-Riemannian model (or analogy). This claim is immediately relevant to our discussion of Curiel's paper. Indeed, when Curiel says that GTR is not in need of an interpretation (meaning: an extra-theoretical model) he seems to imply that the pseudo-Riemannian model of gravity is part of the GTR, that it is sufficient to understand all experiments involving gravitation, that we do not need other models, and even, that other possible models of gravity respecting the same core relational structure are not part of GTR.

If we read Curiel's paper in this way, then his theoretical/extra-theoretical demarcation seems quite *a priori*. Pseudo-Riemannian geometry just is part of GTR, whereas Euclidean models helping us to understand that non-Euclidean geometry better (the way Klein's model helps us to understand Lobachevskian geometry better) are simply excluded. Each interpretation of GTR, apart from the pseudo-Riemannian model, is taken as superfluous, without further ado, even if that interpretation happens to respect the same core relational structure, implying strong empirical equivalence. Whitehead would disagree with Curiel's *a priori* demarcation. For Whitehead, all models or analogies that help us to better understand gravity are legitimate parts of the physical theory of gravitation (at least, if they are adequate and coherent).

But there is more. According to Whitehead, Einstein's geometrical analogy of gravitation is a dangerous one. The geometrical character of the clarifying subject of Einstein's analogy, and the widespread ignorance of the true nature of pure mathematics, may lead us to misconceive the relational essence of gravitational phenomena as if it is all about the shape-iness of a substance, all about the variable curvature of a pseudo-Riemannian manifold. Moreover, it may lead us to identify this substance, this manifold, not only with the gravitational field, but also with spacetime. In fact, it led Einstein and most of his followers to do so. In what follows, I will deal with some of the elements constituting an answer to the question of why, according to Whitehead, the identification of spacetime with a substantial and variably curved manifold is undesirable. For now, I

limit myself to an introductory account of Whitehead's attempt to undo Einstein's geometrical analogy of gravitation, and to separate Riemannian geometry from gravitational physics.

One of the most important steps in the Whiteheadian attempt to separate geometry and gravity is the abstraction of the mathematical pattern underlying Einstein's geometrical analogy of gravitation. This abstraction led Whitehead to identify the purely abstract tensor patterns that form the relational essence of Einstein's theory of gravitation. Consequently, Whitehead gave an exposition of pure tensor theory in Part III of his 1922 book on GTR, *The Principle of Relativity with Applications to Physical Science*. Whitehead's reason for giving a purely abstract presentation of tensor theory instead of referring to an existing presentation, is made perfectly clear in the Preface: "The theory of tensors is usually expounded under the guise of geometrical metaphors which entirely mask the type of application which I give to it in this work" (R vi). Whitehead continues: "For example, the whole idea of any 'fundamental tensor' is foreign to my purpose and impedes the comprehension of my application." Indeed, Einstein's notion of a "*fundamental*" tensor is the notion of a (*geo*)*metric* tensor.

Of course, a next step in the separation of geometry and gravity, after taking the step of isolating the purely abstract relational structure of the geometrical metaphor of gravitation, is the introduction of an alternative, non-geometrical analogy of gravitation, illustrating the same purely mathematical pattern, clarifying the same relational essence of gravitation. As will become clear when I discuss the context of discovery, Whitehead replaced Einstein's analogies—Einstein's discovery analogy, and the resulting pseudo-Riemannian geometrical curvature analogy of gravitation—with the Minkowskian electromagnetic interaction analogy of gravitation. This led him to replace Einstein's *fundamental*, pseudo-Riemannian *metric* tensor (unifying spacetime geometry and gravitational physics) with an *action or impetus* tensor (a gravitational tensor), defined against the background of a separate Minkowskian *metric* tensor (a separate spacetime tensor); and it led him to replace Einstein's *geodesic* paths with *least action* paths. All these closely related replacements were not intended to (even though they slightly did) change the purely abstract relational structure of GTR. They were intended to change the meaning of GTR. They constituted an interpretation switch. Whitehead spoke about his interpretation as "an alternative rendering of the theory

of relativity” in which “the meanings ascribed to the algebraic symbols are entirely different” (*R v*).

To summarize, if we consider “interpretation” as “interpretation by analogy,” then Einstein’s interpretation of GTR in terms of the geometrical analogy of gravitation constitutes Curiel’s theoretical interpretation of GTR, in terms of which all gravitational experiments can be understood, and next to which no extra-theoretical interpretations are needed. However, this implies that Curiel’s position boils down to an *a priori* rejection of the need for Euclidean interpretations of the pseudo-Riemannian geometry to properly understand GTR, as well as an *a priori* rejection of the need for any alternative interpretation of gravitation to improve upon Einstein’s interpretation, e.g., by preventing some of its undesirable side-effects, while trying to respect the relational essence of GTR.

#### *Interpretation by embodiment*

A possible defense of Curiel’s position consists in the rejection of the relevance of the notion of interpretation by analogy when focusing on experiments. Indeed, Curiel might say that, regardless of the question of whether or not analogies did play an important role in the past process of discovering GTR, today they are not needed any longer. If we leave aside the context of discovery, and the initial construction of GTR, to focus on the context of justification, and on the application of GTR to experiments, then the only thing that matters is that our experiments of gravitation embody the core relational structure of GTR, or to put it the other way around, that the core relational structure of GTR appropriately applies to our experiments of gravitation. In other words, Curiel might hold that GTR does not even include Einstein’s geometrical “interpretation by analogy,” and that the only theoretical interpretations it includes are the experimental “interpretations by embodiment,” that is, the purely abstract theory’s applications to experimental situations.

Of course, interpretation by embodiment entails interpretation by analogy, at least if we embody the core relational structure of GTR, not only in the semantic field of experiments, but also in another semantic field (e.g. non-Euclidean geometry), with the purpose of clarifying the former in terms of the latter. But the defense of Curiel’s position offered here clearly holds that the only interpretation by embodiment that is needed in GTR is the experimental interpretation by embodiment. At first sight, this might seem like a good defense. For instance, Suzanne Langer defines

a scientific concept as an abstracted logical form or relational structure, and then offers an interesting definition of interpretation:

Finding applications for concepts is called *interpretation of an abstract form*. . . . If, for instance, we would interpret the abstract concept of ‘rotation,’ we would think of the rolling of a wheel, the motion of a heavenly body, the spinning of a top, the whirl of a propeller. Wheel-rolling, globe-turning, top-spinning, propeller-whirling, are all *interpretations* of the form, *all different contents for the abstract concept* ‘rotation’ . . . Physics deals with any forms which may have physical things for their contents. Biology deals with just those forms that apply to living matter. That is to say, the special sciences take cognizance of all those and only those conceptual patterns, or formulae, to which they can give some *interpretation* relevant to their chosen subject-matter. Interpretation is the reverse of abstraction; the process of abstraction begins with a real thing and derives from it the bare form, or concept, whereas the process of interpretation begins with an empty concept and seeks some real thing which embodies it. (37-38)

Langer gives yet another example of a most relevant scientific concept: “the *concept of oscillation*” (36). She invites us to consider “how many motions follow the general pattern called ‘oscillation,’” and she mentions “the swing of a pendulum, the swaying of a skyscraper, the vibration of a violin-string over which the bow is passing, the chatter of our teeth on a cold day” (35). Again, Langer is in line with Whitehead, who wrote:

In the sixteenth and seventeenth centuries, the theory of periodicity took a fundamental place in science. Kepler divined a law connecting the major axis of the planetary orbits in which the planets respectively described their orbits with the periods: Galileo observed the periodic vibrations of pendulums: Newton explained sound as being due to the disturbance of air by the passage through it of periodic waves of condensation and rarefaction: Huygens explained light as being due to the transverse waves of vibration of a subtle ether: Mersenne connected the period of the vibration of a violin string with its density, tension, and length. The birth of modern physics depended upon the application of the abstract idea of periodicity to a variety of concrete instances. (*SMW* 31)

There is, however, a problem with these quotes from Langer and Whitehead. Their examples do not invite us to consider interpretations in extra-theoretical semantic fields, simply because the semantic field of classical physics is intuitively clear to start with. We can directly observe, or

simply imagine, the application of “rotation” to wheels, globes, tops, and propellers, or the application of “oscillation” to pendulums, skyscrapers, violin-strings, and waves. But the semantic field of GTR is not intuitively clear to start with. We cannot directly observe, or simply imagine, the application of “tensor contraction” to solar systems, galaxies, or black holes.

It is no coincidence that the rise of STR, GTR and quantum mechanics led to an increasing demand for the popularization of physics—popularizers are supposed to offer concrete analogies and metaphors to laypeople in order to increase their understanding of the abstractions of physics (see Whitworth). However, it would be an illusion to think that physicists and mathematicians, as opposed to laypeople, are simply able to think in terms of pure abstractions without the help of analogies and metaphors. Einstein’s use of tensors, and Whitehead’s use of a purely abstract logic of relations, presented a real challenge to the imagination of physicists and mathematicians. According to Lord Haldane in *The Reign of Relativity*, by using tensors and by using logic, “Einstein . . . chastised the physicists with whips,” and “Whitehead . . . chastised the mathematicians with scorpions” (74).

Even physicists dealing with GTR need to interpret the tensor operations in Einstein’s equations in more intuitive contexts than the context of application of GTR. This is why they usually interpret these tensor operations and equations in terms of multi-directional curvatures of, and geodesic paths in, variably curved two-dimensional surfaces. Some physicists and mathematicians have indeed claimed an ability to imagine higher-dimensional generalizations, but it is quite certain that they took recourse to two-dimensional cases as well. For example: “George Boole’s daughter, Alicia Boole . . . had a remarkable ability to ‘see’ in four dimensions” (Gray 348). Of course, she did not claim to have access to the fourth dimension in the way some paranormal mediums did, but it is unclear what “seeing” in four dimensions actually amounts to. Even if a talent for higher-dimensional visualization exists, it does not weaken but rather strengthens my contention that scientists always need to interpret the relational structures of GTR in some intuitive context—whatever its dimensionality. Even pure mathematicians are not born in a Platonic heaven of pure forms. However, whereas the aim of the pure mathematician is the construction of pure logical forms without embodiment, of pure potentials without actuality, applied mathematicians and physicists are ultimately interested in embodied forms, in actualized

potentiality; and in the latter case, the absence of an intuitive embodiment is a serious handicap.

But there is more. Not only would we handicap physicists when restricting their liberty of imaginative interpretation in terms of clarifying analogies and metaphors, we would also miss out on an important criterion to justify their use of a particular set of abstractions. Indeed, even in the context of justification there is more at play than the criterion to mathematically represent well-known and innovative experimental set-ups—more than the requirement to be able to theoretically interpret a whole range of experimental systems. I will call this additional criterion the intelligibility criterion. It is the requirement to provide an intelligible embodiment of the purely formal theories of contemporary physics.

### *Models of intelligibility*

Prior to introducing Whitehead's intelligibility criterion for contemporary physics, let us look at a very rough sketch of the history of the dominant intelligibility criteria in physics. In the initial stage of modern physics, exemplified by Galileo's pioneering work, physical intelligibility meant being able to design a machine that models the physical phenomenon under scrutiny—a mechanical device an artisan could construct. However, in this early age of mechanical philosophy, designing a mechanical model boiled down to giving a particular geometrical description. Galileo's "language in which the book of nature was written" was "the language of mechanics," which actually was "a comparative, relativized geometry of ratios" (Machamer 65). Hence, saying that in this age of the machine metaphor, the mechanical model provided the model of intelligibility is also saying: "the *more geometrico* provided the model of intelligibility and proof for science" (67). But it is important to stress: "The geometry involved was not a pure geometry but the physical geometry of the mixed sciences. It was the geometry of Archimedes, the geometry of proportions and of the proportions of machines considered relative (or in relation to) one another" (67).

Noam Chomsky describes this first period in the history of physics (also by reference to Peter Machamer) as follows:

'The mechanical philosophy' [is] the idea that the natural world is a complex machine that could in principle be constructed by a skilled artisan. "The world was merely a set of Archimedean simple machines hooked together," Galileo scholar Peter Machamer observes, "or a set of colliding corpuscles that obeyed the laws of mechanical

collision.” The world is something like the intricate clocks and other automata that excited the scientific imagination of that era, much as computers do today.

Adopting the mechanical philosophy, “Galileo forged a new model of intelligibility for human understanding,” Machamer argues plausibly, with “new criteria for coherent explanations of natural phenomena” based on the picture of the world as an elaborate machine. For Galileo, and leading figures in the early modern scientific revolution generally, true understanding requires a mechanical model, a device that an artisan could construct.” (49-51)

With Newton’s action-at-a-distance theory of gravitation, in violation of the early philosophy of the natural sciences, the mechanical model of intelligibility ran into trouble, and ultimately collapsed. As exemplified by Newton to start with, and by Maxwell to end with, for quite a while physicists still felt that when mechanism fails, understanding fails. However, thanks to the development from ancient to algebraic geometry, and the addition of infinitesimal analysis, geometry still provided a means of describing physical systems, including action-at-a-distance systems. The failure of mechanical explanation did not imply the failure of geometrical description—geometry emancipated from mechanics. The continuing failure of mechanical understanding (e.g., the failure to explain gravitational, as well as electric and magnetic phenomena in terms of a mechanical ether), and the increasing explanatory success of geometry (e.g., Faraday’s success in explaining electromagnetic induction phenomena in terms of geometrical lines of force), led physicists to separate the mechanical criterion of intelligibility and the geometrical criterion of intelligibility, and to give priority to the latter above the former. Whereas we might call the initial stage of physics the age of the machine metaphor, we might call this second stage of physics the age of the geometry metaphor.

Finally, with the rise of relativity and quantum physics, a third period in the history of physics dawned. In the second period, geometry emancipated from mechanics (mathematics emancipated from the mathematics of the mixed sciences); in the third period, pure mathematics emancipated from geometry (from the semi-intuitive, semi-abstract mathematics of the middle period). The use of tensor theory in GTR, and Hilbert space theory in quantum mechanics, are only two of the many examples that can be given to illustrate the evolution towards the use of more and more abstract mathematics in physics. This leaves us with three options

to answer the main question of what this evolution might mean for the choice of a criterion of intelligibility.

The first option is to say that pure mathematics should replace geometry as the model of intelligibility for physics. But this option is based on a myth, the myth of a purely abstract understanding of nature. As I already said, not even pure mathematicians are born in Plato's heaven. Pure mathematics is a movement from concrete totality to abstract structure, and applied mathematics a movement in the opposite direction. Pure mathematics aims at constructing abstract and potential types of order, but physics and philosophy aim at understanding concrete and actual togetherness. The "error of mistaking the abstract for the concrete" is what Whitehead has called the "Fallacy of Misplaced Concreteness" (*SMW* 51). Avoiding this error here is being aware that even when mathematical abstractions are applied to our understanding of nature, they are throwing *only* a partial light on *only* the structural aspects of the concrete patterns nature displays. Mathematics falls short of actuality. In a dialogue with Lucien Price, the old Whitehead said:

Mathematics is essentially a study of types of order. In its earlier forms it had to do with number and quantity. That is its historical origin; the idea of mathematical logic is comparatively modern. But while mathematics is a convenience in relating certain types of order to our comprehensions, it does not, as used to be supposed, give us any account of their actuality. (*D* 214)

What used to be supposed—e.g., by Pythagoras—is "that the mathematical entities, such as numbers and shapes, were the ultimate stuff out of which the real entities of our perceptual experience are constructed" (*SMW* 27). But for Whitehead, pure logical subjects (variables) play the leading role in mathematical propositions, not real entities. In retrospect the Pythagorean dream might seem "crude, and indeed silly" (*SMW* 27). But what can still be upheld is that mathematics as a human practice is instrumental to acquire a partial understanding of the structural connectedness of real entities.<sup>6</sup>

The second option consists in saying that we should save the geometrical intelligibility criterion, and look for the appropriate geometrical embodiment of the pure mathematics that haunts physics. I hold that Einstein took this second option, even though he did not make an *ad hoc* decision, but was naturally led to a geometrical interpretation of gravitation by the main analogy guiding his discovery of GTR (to be discussed

later). I also hold that the practice of gathering all the mathematics required by physicists under the hat of geometric algebra, and then using this semi-intuitive, semi-abstract geometric algebra in all branches of physics—a practice I recognize in David Finkelstein, David Hestenes, and Roger Penrose—is another contemporary instance of choosing the geometrical intelligibility criterion as an important intelligibility criterion for physics. Actually, the history of geometric algebra goes back to Grassmann and Clifford and to Whitehead's 1898 *UA*. (see Hestenes 1-2). However, what I question here is not the use of multiple intelligence criteria, but the absolute priority of the geometrical criterion, especially in GTR.

The third option is to introduce a new model of intelligibility for physics, pushing both the machine model (which failed us, but still attracts us) and the geometry model (which still has its merits) from the position of dominant intelligibility model. The change of intelligibility model does not exclude older models from still playing, next to the new model, a legitimate role in support of our overall understanding, even though a less important and more restricted one. Whitehead—despite the dominance of geometry in his mathematical career, and its obvious importance for the complex development of the notion of extension throughout his philosophical career—took this third option.

#### *Whitehead's model of intelligibility*

Whitehead introduced a new model of intelligibility for physics as well as philosophy. We may say that Whitehead replaced the machine metaphor with the organism metaphor, and the mechanical philosophy of science with an organic philosophy of science. For example, using “organism” as a metaphor, Whitehead conceived of the enduring entities studied by particle physics as small-scale “organisms.” However, Whitehead's philosophical analysis did not stop at enduring entities, and Whitehead's attention was mainly, and increasingly, focused on one particular type of organism—the type he identified when reading William James and which became paradigmatic for his overall philosophy. Even though they are not usually called organisms, I am referring to the drops of experience constituting each human stream of experience. In day-to-day language, these Jamesian drops of experience are called “moments of experience,” even though the meaning of the word “moment” in “a moment of experience” is not to be confused with the notion of an instantaneous moment of time—a moment of experience definitely has spatio-temporal extension.

In principle, the conception of moments of experience as spatio-temporal quanta, constituting a continuous stream of experience, immediately requires us to give a Whiteheadian account of the interplay of causal and spatio-temporal connectedness, internal and external relations, occurrence and recurrence and endurance, discontinuity and continuity, and so forth. Even Whitehead's critique of our habit to overemphasize the abstract analysis of time in terms of instants (instead of durations) would require us to evoke many of his sources, e.g., Bradley's *The Principles of Logic* (52-56), and James' *The Principles of Psychology* (605-42), as well as many philosophical similes, e.g., Bergson's intuition of real duration and Husserl's phenomenology of internal time consciousness.

In practice, of course, I cannot engage in giving such an account here, and I merely reiterate a quote from Whitehead's 1922 book on GTR—a book in which only an incomplete account of his philosophy of experience is given, for it is pushed into the background to be more fully articulated in his later works:

In testing . . . the theory . . . we cannot wisely confine ourselves to the consideration of a few happy applications. The history of science is strewn with the happy applications of discarded theories. There are two gauges which every theory must pass. There is the broad gauge which tests its consonance with the general character of our direct experience, and there is the narrow gauge which is . . . the habitual working gauge of science. (*R* 3-4)

For Whitehead, next to the empirical adequacy requirement of experimental interpretation, each physical theory has to meet the intelligibility requirement of experiential interpretation. It is important to realize that the latter requirement is of a different nature than the former. To interpret an experiment is to be able to apply the pure mathematics of the physical theory in order to describe and explain it, to produce and predict it, and by doing so, to actualize a pure potential, to embody a purely abstract relational structure. However, to interpret a moment of experience is to test the theory's "consonance with the general character of our direct experience." It is not about strictly applying the theory to describe and explain, to produce and predict non-experimental moments of experience. It is about the coherence of the theory with "the general character of our direct experience," where Whitehead's expression "direct experience" mainly refers to "sense-perception," and where "in practice sense-perception is narrowed down to visual perception" (*PR* 36). Whitehead's intelligibility

requirement is the requirement of rational coherence with the general character of our experience—with a special emphasis on our immediate visual displays of the present.

*The entanglement of physics and philosophy*

The articulation of the general character of our experience involves the kind of (physiology and psychology inspired) philosophical analysis of experience I did not give, but alluded to, above. In other words, the introduction in the context of justification of the Whiteheadian intelligibility criterion—a rational coherence criterion—next to the empirical adequacy criterion of successful applicability and experimental exemplification, implies a mixture of philosophy and applied mathematics which is likely to repel both physicists and philosophers. No wonder that Whitehead, immediately after the last quote from *The Principle of Relativity* given above, tells us the story of two distinguished persons to whom he explained, prior to publication, the scheme of his 1922 book:

The philosopher advised me to omit the mathematics, and the mathematician urged the cutting out of the philosophy. At the moment I was persuaded: It certainly is a nuisance for philosophers to be worried with applied mathematics, and for mathematicians to be saddled with philosophy. But further reflection has made me retain my original plan. The difficulty is inherent in the subject matter. To expect to reorganise our ideas of Time, Space, and Measurement without some discussion which must be ranked as philosophical is to neglect the teaching of history. (4)

The teaching of history can be made explicit by means of my rudimentary sketch of the evolution of the dominant intelligibility criteria in physics. With each intelligibility criterion we effectively buy into a philosophy. Most revealingly, the machine metaphor was packaged with a large number of philosophical presuppositions, consciously or unconsciously entertained. The machine model of intelligibility was part and parcel of a philosophy that largely survived the collapse of the model itself, and still animated generations of physicists celebrating the geometry model of intelligibility. This philosophy has been given many names. For example, Whitehead called it “scientific materialism” (*SMW* 17), and Dijksterhuis called it “the mechanized world view.”

The permanent intertwining of the physicist’s intelligibility criteria with his or her overall philosophy, often tacitly in the background, implies that with the introduction of the organic metaphor, or more specifically, with

the introduction of the human experience metaphor (i.e., the intelligibility criterion of rational coherence with the general character of human experience), Whitehead has not introduced a previously non-existing type of mixture. The mixture of philosophy and physics has always played an explicit role in times of initiating or reorganizing basic concepts of physics, shifting into oblivion in times of relative stability of those concepts, while remaining normative in the physicists' intelligibility criteria.

According to Whitehead, the mixture of philosophy and physics even involves the essence of its ingredients:

The study of philosophy is a voyage towards the larger generalities. For this reason in the infancy of science, when the main stress lay in the discovery of the most general ideas usefully applicable to the subject-matter in question, philosophy was not sharply distinguished from science. To this day, a new science with any substantial novelty in its notions is considered to be in some way peculiarly philosophical. In their later stages, apart from occasional disturbances, most sciences accept without question the general notions in terms of which they develop. The main stress is laid on the adjustment and the direct verification of more special statements. In such periods scientists repudiate philosophy. (*PR* 10)

In the later stages of Newtonian physics, for example, many physicists were satisfied with Newton's physical principles, and disclaimed metaphysics. However, Whitehead writes: "The fate of Newtonian physics warns us that there is a development in scientific first principles," and he adds that this development requires "interpretations of meaning" (*PR* 10). The introduction of Einstein's principles of relativity and equivalence, and Planck and Bohr's quantum of action, destabilized physics, initiated significant reorganizations, and reanimated the philosophical quest for meaningful concepts.

Of course, one might say that by now GTR is a stable theory, and that Curiel is only a typical exponent of the current, stable period in the life of GTR, repudiating all discussions of the appropriate philosophical interpretation as irrelevant. To a certain extent this is correct, and I can even add that the lack of success of Whitehead's alternative interpretation of GTR is mainly due to the fact that it was *ad hoc*, and when published in 1922, GTR was already too stable a theory, even for the initially very critical British physicists, in order for them to be open to renegotiate or even change its by then accepted geometrical interpretation. In this sense, Whitehead's theory came "out of season for the physicist" (Lowe 127). On

the other hand, since the rise of quantum mechanics, the concern for an overarching philosophical interpretation of both relativity and quantum physics has been vivid and permanently at the center of attention of both philosophers and physicists. And in view of the history of physics, and of the challenge of quantum mechanics, our attitude towards alternative philosophical interpretations of GTR should not be motivated by the question: “how to get rid of philosophical interpretations in the field of GTR?” But rather: “what is an appropriate philosophy for contemporary physics?”

Unfortunately, most physicists still do not get the question on the philosophy-physics relationship right. They are selectively deaf to what the founders of physics they admire have always proclaimed—that philosophy is important for physics, especially in times of innovation and reorganization. Einstein, for example, urges his colleagues to be aware of the role of philosophy in physics. His phrasing is similar to Whitehead’s, and he even holds that the role of philosophy in physics is too important to leave it up to philosophers who are not physicists:

It has often been said, and certainly not without justification, that the man of science is a poor philosopher. Why, then, should it not be the right thing for the physicist to let the philosopher do the philosophizing? Such might indeed be the right thing at a time when the physicist believes he has at his disposal a rigid system of fundamental concepts and fundamental laws which are so well established that waves of doubt cannot reach them; but, it cannot be right at a time when the very foundations of physics itself have become problematic as they are now. At a time like the present, when experience forces us to seek a newer and more solid foundation, the physicist cannot simply surrender to the philosopher the critical contemplation of the theoretical foundations; for, he himself knows best, and feels more surely where the shoe pinches. In looking for a new foundation, he must try to make clear in his own mind just how far the concepts he uses are justified, and are necessities. (*Ideas* 290)

Like Whitehead, Einstein notices distinctions between innovative and well-established concepts and laws and between reorganizing and reaffirming periods in the history of physics—distinctions Thomas Kuhn has turned into the opposition between revolutionary and normal science. Like Whitehead, Einstein makes clear that at least as long as there is no solid foundation for the whole domain of physics, including relativity and quantum physics, we are in need of philosophical interpretation.

And yet, even one of the main supporters of Whitehead's theory of gravitation in the 1950s, John L. Synge, did not realize that its main purpose was to offer a philosophical interpretation of GTR, and not just an alternative physical theory meant to compete with Einstein's in the arena of experiment. Synge considered Whitehead's philosophical interpretation to be "only a wrapping for physical theory," and took "a savage joy in tearing off this wrapping and showing the hard kernel of physical theory concealed in it" (9). Consequently, in Synge's 1951 lectures on Whitehead's theory, "the philosophy is discarded and attention is directed to the essential formulae" (9). This implies that Synge has done for Whitehead's theory what Curiel seems to advise for Einstein's GTR: Consider only the purely abstract core of GTR as it applies to experiment, and reject all further interpretation by embodiment, especially all wrapping of the purely formal kernel of GTR in a philosophical integument such as Whitehead's philosophy of experience. I disagree with Synge and with Curiel.

*The bifurcation of nature*

Apart from the reasons already offered—the historical argument, and the contemporary need to reorganize the whole conceptual foundation of physics, including GTR—an additional reason for my disagreement with Synge and Curiel is that the rejection of a new intelligibility criterion, and the rejection of a new guiding philosophy for physics, only leads to a conscious or unconscious relapse to the scientific materialism presupposed by the older machine and geometry models of intelligibility. Consequently, it leads to an undesirable "bifurcation of nature" (CN26)—the bifurcation of nature into the primary, objective, and "true" world of the mechanized and geometrical patterns of the externally-related material substances of physics, and the secondary, subjective, and apparent world arising from the internally-related moments of human experience.

Scientific materialism entails the bifurcation of nature and its many dualisms—machine/man, nature/culture, embodied/pure, matter/mind, primary/secondary, objective/subjective, truth/appearance, explanation/interpretation, etc. Here I give only one example to illustrate how deeply rooted the bifurcation idea is, even in the minds of contemporary physicists, and then highlight that Whitehead's intelligibility criterion, his requirement of rational coherence with the general character of experience, was aimed at overcoming exactly the pervasive and many-headed incoherence implied

by modern physics—the bifurcation of nature. The example is provided by a very revealing quote from the first chapter of one of the bestsellers of Brian Greene, *The Fabric of the Cosmos*. Greene writes:

It's easy to be seduced by the face nature reveals directly to our senses. Yet, . . . I've learned that modern science tells a very different story. *The* overarching lesson that has emerged from scientific enquiry over the last century is that human experience is often a misleading guide to the true nature of reality. Lying just beneath the surface of the everyday *is* a world we'd hardly recognize. . . . I'm referring to the work of ingenious innovators and tireless researchers—the men and women of science—who have peeled back layer after layer of the cosmic onion, enigma by enigma, and revealed a universe that is at once surprising, unfamiliar, exciting, elegant, and thoroughly unlike what anyone ever expected. . . . The insights of modern physics have persuaded me that assessing life through the lens of everyday experience is like gazing at a van Gogh through an empty Coke bottle. Modern science has spearheaded one assault after another on evidence gathered from our rudimentary perceptions, showing that they often yield a clouded conception of the world we inhabit. . . . By deepening our understanding of the true nature of physical reality, we profoundly reconfigure our sense of ourselves and our experience of the universe. (5)

Greene clearly opposes our direct experience to our scientific research—the former being rudimentary, superficial, clouded, and misleading, the latter being innovative, deep, insightful, and “true.” By doing so, he bifurcates nature into the familiar world of our direct experience, and the unfamiliar world of our scientific research. Greene says that it is easy to be seduced by the former world, but he obviously tries to seduce us into accepting the latter world as the “true” world, which the ingenious and tireless heroes of science have conquered in their never-ending war against the “illusions” of our direct experiences. Greene’s strategy of seduction illustrates a common strategy in bifurcating descriptions—one that has clearly been identified and articulated by Isabelle Stengers:

The seduction of the idea of a truth that should hurt and disenchant, which should go beyond illusions and destroy them, is exemplified each time a scientist or somebody speaking in the name of science promotes a version of the bifurcation of nature. And this seduction may well explain the very stability of the bifurcation theme. Be it when Galileo rejoiced in making the experimental fact (dealing only with the way heavy bodies fall in a frictionless world), the

possibility and power of which he had just discovered, the ground for expelling philosophers and theologians from the new territory of science. Be it when Jacques Monod deduced from molecular biology and Darwinian selection the existential loneliness of humanity in a meaningless universe. Or be it each time thinking and feeling are reduced, in the name of science, to the blind interplay of neurons: What triumphs is emphatically not scientific objectivity but a strong affective association of truth with conflict and war. (245)

According to Stengers, Whitehead's philosophy of experience may be defined as an antidote "against our fascination with the power of truth," as an "open set of propositions," constituting "a peace-producing philosophy," directed "against the value our Western tradition attributes to conflict as the very mark of truth" (244-45). She correctly identifies Whitehead's main philosophical motivation as the urge to think things together, not apart; to pursue coherence and adequacy, not bifurcation and reduction. And Stengers gives a most appealing description of what a peace-producing or constructive philosophy such as Whitehead's is all about:

Creating togetherness is an ideal seeking satisfaction, and to renounce it would be to renounce the adventure of hope. To try to think together, while knowing that we are, should be, and must continue to present ourselves as unable to transcend the actual limitations of this togetherness or to escape to some dreamed of universality, is, I would submit, the very stamp of a constructive philosophy. Speculation thus becomes not the discovery of the hidden truth justifying reality, but a crucial ingredient in the construction of reality. (239)

Stengers calls her own response to the ideal of a peace-producing philosophy "cosmopolitics," and she adds: "cosmopolitics defines peace as an ecological togetherness, where 'ecological' means that the aim is not toward a unity beyond differences, which would reduce those differences through a goodwill reference to abstract principles of togetherness, but toward a creation of concrete, interlocked, asymmetrical, and always partial graspings" (248-49). Stengers' almost poetic expression of the ideal, does not lead her to blind optimism, not with respect to her own cosmopolitics, and not with respect to Whitehead's philosophy. In fact, she holds that Whitehead himself was too optimistic when thinking that the internal development of physics might by itself lead to the disappearance of the bifurcation of nature into the world of scientific experiments and facts, and the world of human experiences and opinions:

When Whitehead was writing *Science and the Modern World*, he was enjoying the hope that the epoch when the sciences sided with the bifurcation of nature was about to be closed. We know that he was overly optimistic in this matter: Not only is the reductionist stance still dominant, but we have very good reasons *not* to believe, as he did, that scientific innovation as such might endanger it. The idea that science is at war with opinion, that its very advance means “progress” framed as, “everyone thought such and such before, but we (scientists) know that, . . .” has proved stronger than all the revisions of what scientists may indeed claim to know. In other words, I would state that we must now accept the fact that the field of science needs a still more concrete description of the values it produces: Not only its epistemological values, but also its political values reproduce an opposition between a “rational,” objective grasp of reality and an “irrational,” subjective, culturally embedded opinion. As long as the “science against opinion” image is patiently accepted, infecting scientists and nonscientists alike, the bifurcation of nature will be produced again and again as both the condition for science and for its confirming result. (249)

This is not the place to further Stengers’ analysis of how the political values of science, next to the ontology and epistemology of scientific materialism, sustain the bifurcation of nature, and how cosmopolitics—as the term suggests—tries to take the extra, political dimension into account. However, bringing in Stengers has shown that the quote from Brian Greene is not an isolated case, but representative of the fact that the bifurcation view is not merely something of the remote past; it is a vision still dominating the present. Bringing in Stengers has also shown that the Whiteheadian attempt to make contemporary physics and human experience cohere can still inspire contemporary philosophers to try to counter the dominance of the bifurcation view. Whitehead’s proposal of the human experience metaphor to guide physics and to promote coherence, as well as his more general proposal of analogical interpretation as an indispensable means to add meaning to science, still stand.

When Greene compares our direct experience of reality with gazing through an empty bottle at a van Gogh painting, he furthers his claim that direct experience is misleading by means of an analogy that appeals to our direct experience of gazing through an empty bottle. The Whiteheadian claim is that we do not only need analogies appealing to our direct experience when we consider the limitations of direct experience, we also need them in our scientific exploration of reality. And to Greene’s requirement

to reconfigure our sense of ourselves, and our experience of the universe, in the light of science, we oppose Whitehead's requirement to reconfigure and recontextualize science in order to contrast and harmonize it with our sense of ourselves, and our experience of the universe. If it were possible to ban all analogies from physics, and all recourse to human experience, we would be left with a meaningless science of completely abstract symbols. On the other hand, if it were possible to intensify our practice of analogical interpretation in physics, and to conform to Whitehead's intelligibility requirement, a more meaningful science, occupying its proper place in the overall human adventure, might appear at the horizon of our cultural landscape.

*Whitehead's rejection of the Mach-Einstein interpretation and ontology*

So far I have only addressed Curiel's initial question with respect to GTR: "What sort of interpretation is required, if any?" Curiel's answer is that the maximum we need is Einstein's geometrical interpretation of our experiments of gravitation, and the minimum we need is the application of the pure mathematical formalism of GTR to its experimental context. Whitehead's answer, contrary to Curiel's, is that we need all the interpretation we can get to understand gravitation, at least, as far as the requirements of experimental adequacy and experiential coherence are satisfied.

Curiel does not think that his answer is endangered by the consensus among contemporary philosophers of science that an ontological interpretation of GTR is required in order to counter Einstein's Hole Argument. The Hole Argument is all about the diffeomorphic freedom of the solutions of Einstein's field equation, and the alleged indeterminism this freedom seems to imply. However, according to Curiel, the theoretic freedom is eliminated in the practice of representing experiments, and the diagnosis of indeterminism presupposes a hard to defend reading of Einstein's field equation, namely, that it expresses a causal statement of the form, "ponderable matter determines the structure of spacetime."

When Einstein formulated his generally covariant field equation in November 1915, he definitely assumed that his field equation expressed how ponderable matter (how the energy tensor) determines the structure of spacetime (the metric tensor). In Einstein's Machian reading, ontological priority is attributed to ponderable matter, and ontological relativity to spacetime. So this reading of Einstein's field equation—leading to the Hole Argument according to Curiel—already presupposes an ontological

interpretation, where “already” means “prior to the rise of the request to overcome the Hole Argument by means of an appropriate ontology.” The Hole Argument seems to rest on the assumption that the structure of spacetime is built by relations between material objects; this seems to imply an indeterminism in regions devoid of such objects, which is overcome by holding that points of spacetime cannot be individuated and identified apart from their metrical properties.

Curiel, while accepting the purely formal diffeomorphic freedom, rejects both the assumption on which he thinks the indeterminism argument rests, and the need for an ontological interpretation to neutralize this argument. Whitehead would agree with Curiel’s rejection of Einstein’s Machian interpretation, but not with Curiel’s aim to reject the need for any ontological interpretation.

Whitehead had good reasons to reject Einstein’s Machian interpretation. When Whitehead learned about GTR in September 1916, Einstein’s Machian interpretation was already weakened by the devastating critique of Willem de Sitter, brought to the fore in the context of de Sitter’s discussions with Einstein on the cosmological models GTR could supply. Moreover, Whitehead stressed that the acceptance of the Machian interpretation (that ponderable matter determines the structure of spacetime) leads to circularity. Indeed, as the energy tensor includes the actual spatio-temporal distribution of ponderable matter, its appearance in Einstein’s field equation already presupposes the structure of spacetime it is supposed to determine by means of that equation. Let me express the same issue differently. In order for the energy tensor to function as the input to calculate the gravitational field by means of the field equation, we first have to determine by spatio-temporal measurement the actual spatio-temporal distribution of ponderable matter. However, since Einstein holds that gravitational field and spacetime metric are one and the same entity, this measurement requires knowledge of the actual gravitational field before it is possible (see *R* 83). Again, solving Einstein’s field equation presupposes knowledge of the solution before it is possible.

Notice that Whitehead did not blame causality as such for the circularity that each instance of solving the equation presupposes its actual solution. The real issue is not that the energy tensor is supposed to causally determine the gravitational field. The real issue is that the latter is identified with the spacetime metric, effectively implying a Machian-type interpretation that the energy tensor causally determines the spacetime

metric. According to Whitehead, an appropriate differentiation of gravitational field and spacetime metric can prevent the circularity, for then presupposing knowledge of the actual spacetime metric, prior to solving Einstein's field equation, does not imply presupposing knowledge of the actual solution of the equation, which is the actual gravitational field, and not the actual spacetime metric.

Also notice that Einstein himself did not stick to his Machian interpretation of GTR, even though he did stick to his identification of gravitational field and spacetime metric. In a letter of February 2, 1954, to Felix Pirani, Einstein highlights a circularity similar to the one that Whitehead highlighted in 1922. On people who hold fast to the Machian interpretation, Einstein writes: "Aber sie denken, daß das Feld *völlig* bestimmt sein soll durch die Materie. Dies is aber eine heikle Sache, da die  $T_{ik}$ , welche die 'Materie' darstellen sollen, immer schon das  $g_{ik}$ -Feld voraussetzen" ("Letter" 1-2). ("But they think that the field should be *completely* determined by matter. This, however, is a tricky thing, since the  $T_{ik}$  [the energy tensor], which should represent 'matter,' always already presupposes the  $g_{ik}$ -field [the gravitational field].")

Curiel reads Einstein's field equation as expressing a kind of mutual harmony between energy tensor and metric tensor, rather than a causal influence from energy to metric, and he seems to presuppose that his reading is not in need of any interpretation. But Whitehead would say that every reading already *is* an interpretation, and that an appropriate ontological discussion can lead to further clarification of that interpretation. Whitehead was aware of the fact that his rejection of the Machian interpretation of Einstein's field equation at once requires the rejection *and* replacement of the corresponding Machian ontology. And I can add: so also for Einstein. Both men were aware of the fact that questioning the Machian interpretation entails an ontological discussion. In the already quoted letter to Pirani, e.g., Einstein writes that one should no longer speak of the Machian interpretation because: "Es stammt aus der Zeit, in der man dachte, das die 'ponderabeln Körper' das einzige physikalisch Reale seien" ("Letter" 3). ("It stems from the time in which it was thought that 'ponderable bodies' are the only physically real entities.")

I will not give any detailed account of Whitehead's alternative ontology here, and limit myself to a very short comparison. In Einstein's original Machian ontology, matter is fundamental, and spacetime metric

and gravitational field are derivative and equal. In Whitehead's ontology, actual occasions are the ultimately real entities. Spacetime metric, matter, and gravitational field, are all abstracted from the relatedness of actual occasions. Also, spacetime metric and gravitational field are not identical. The former expresses uniform relations of extension, the latter contingent relations of causality, and like all physical fields, the contingent gravitational field is expressed against the uniform spacetime background. Accordingly, Whitehead writes: "it is inherent in my theory to maintain the old division between physics and geometry. Physics is the science of the contingent relations of nature and geometry expresses its uniform relatedness" (*R v-vi*).

There is one item that remains to be discussed: the claim that points of spacetime cannot be individuated and identified apart from their metrical properties—a claim that arose to neutralize the Hole Argument. Even though Curiel holds that part of the force of the Hole Argument is the Machian interpretation of GTR, and even though Einstein did not stick to the Machian interpretation later in life, Einstein did hold fast to the above claim, which implies that spacetime cannot exist as a non-metrical topological spacetime. In the same year as his letter to Pirani, Einstein wrote:

If we imagine the gravitational field, i.e., the functions  $g_{jk}$ , to be removed, there does not remain a space . . . but absolutely *nothing*, and also no "topological space." For the functions  $g_{jk}$  describe not only the field, but at the same time also the topological and metrical structural properties of the manifold. . . . There is no such thing as an empty space, i.e., a space without field. Space-time does not claim existence on its own, but only as a structural quality of the field. (*Ideas 375*)

How does this relate to Whitehead's ontology? Whitehead shares with Einstein the view that there is no such thing as an empty space, but for Whitehead "an empty space" means "a space without actual occasions." Actual occasions are the ultimately real entities, whereas spacetime is relative. If we imagine the actual occasions, and hence their extensive relatedness, to be removed, there does not remain a spacetime, but absolutely nothing, and also no topological spacetime. However, given an arbitrary nexus of actual occasions, Whitehead leaves open the metaphysical possibility that their extensive relatedness is merely topological, without being metrical.

*Einstein's practice of discovery*

Curiel does not believe that interpretation efforts with regard to GTR will help physicists to discover a more fundamental theory of quantum gravity, harmonizing the general relativistic and the quantum approach. He advances two reasons for this claim. First, it is unlikely that a philosophical renegotiation of the basic concepts of a less fundamental, and to be replaced, theory of gravitation will shed light on the conceptual basis of a more fundamental, and to be discovered, theory. And secondly, it is unlikely that such a philosophical effort with respect to a theory that is extremely successful from an empirical point of view will lead to the discovery of a new theory. Discovery in physics is mainly driven by adverse experimental data.

I think Einstein would firmly disagree with Curiel. Look, e.g., at (a simplified version of) Einstein's discovery of STR. The main impetus that led to the discovery of STR was Einstein's conceptual unease with an empirically highly successful theory—Maxwell's theory of electric and magnetic fields. Einstein was repelled by the idea that, depending on the inertial reference frame chosen for the application of Maxwell's laws, one and the same phenomenon was represented by different mixtures of electric and magnetic fields. If Einstein had acted according to Curiel's view, he should have said: "the non-uniqueness of the electric-magnetic mix does not prevent me from successfully calculating the relevant experimental data—so what?" Einstein, however, did look for a way out, and Lorentz's publications on Maxwell's theory were a main source of inspiration.

Lorentz's way out was to say that the electric-magnetic mix in a preferred inertial reference frame—a frame at rest in the ether—is the true mix, and each mix in a reference frame uniformly moving compared to the preferred one is only apparent. However, the failure to detect the ether—or better: movement compared to the ether—led Lorentz to also consider apparent times and apparent lengths, next to a true time and true lengths. For Einstein, Lorentz's way out, implying true and apparent electric-magnetic and length-time mixtures, was unacceptable.

Einstein's own way out was to conceive of electricity and magnetism as abstractions from a deeper reality—electromagnetism—and to reject the notion of a preferred reference frame. Einstein aimed at a uniform description of electromagnetic phenomena—uniform across all inertial reference frames, made equivalent by the abolition of a preferred one. Einstein's requirement of a frame independent physics

of electromagnetism, in a sense, was already satisfied by the Maxwell-Lorentz theory, if only, when changing from one to another inertial reference frame, one conceived the changing electric-magnetic mix, and the changing space-time mix, as frame dependent splits of frame independent realities—electromagnetism and spacetime. However, Einstein's requirement of a corresponding, frame independent mechanics, manifesting the same frame dependence of time and length, led him to generalize Newtonian mechanics to relativistic mechanics.

In other words, Einstein's conceptual unease with the non-uniformity of the Maxwellian description across inertial reference frames ultimately led him to unify electricity and magnetism, and space and time, and to discover an electromagnetic physics, and a spacetime mechanics, of which the general concepts and laws are frame independent or absolute, and of which the specific representations in terms of electric and magnetic fields, and in terms of space and time coordinates, are frame dependent or relative. Of course, only after Minkowski's involvement, the Einsteinian unification found its proper mathematical expression in terms of tensors. Today, in STR, the Faraday tensor (or the closely related Maxwell tensor; see Schwarz and Schwarz 171-75) expresses the electromagnetic field, and electric and magnetic vector fields are no more than frame dependent vector components of the electromagnetic tensor field. Also, the Minkowskian metric tensor is STR's blueprint of all spacetime intervals, and distances and durations are no more than frame dependent space and time components of spacetime intervals.

My conclusion is that Einstein's discovery of STR was mainly driven by conceptual reflection, and not mainly by adverse experimental data (even though, e.g., the unsuccessful experiments to determine the earth's motion relative to the ether did play a role). Moreover, I think the same conclusion holds for GTR. In fact, GTR does not only illustrate the importance of conceptual reflection with regard to existing theories that are not endangered by adverse experimental data, but also the importance of analogies for conceptual reflection guiding discovery. For this reason, let us focus on the main impetus that led to Einstein's discovery of GTR—the analogy behind Einstein's equivalence principle.

The STR had two major shortcomings. It dealt with Maxwell's theory of electricity and magnetism, and with Newton's mechanics, but it left out Newton's theory of gravitation. In Newton's law of gravitation, the force of gravitation is expressed in terms of (the inverse square of) the

distance between masses, whereas STR requires frame independent laws, expressed in terms of frame independent spacetime intervals instead of frame dependent distances. Also, in Newton's law of gravitation, time is absent, implying an instantaneous gravitational interaction between masses at a distance. But the STR requires all interactions in nature to be field-like interactions, propagated at limited speeds, not exceeding, but possibly equal to, the speed of light in vacuum. The response to this first shortcoming of STR was the search for a special relativistic theory of gravitation. A second shortcoming of STR is that its frame independence principle—the principle of relativity—is a *special* one, only referring to inertial or uniformly moving reference frames, leaving out accelerated reference frames. The response to this second shortcoming of STR was the search for the appropriate extension of STR to a *general* theory of relativity in which the laws of nature are frame independent, not only with respect to inertial reference frames, but also with respect to accelerated reference frames.

In 1907, while attempting to modify Newton's law of gravitation to make it fit in STR, Einstein had what he called the "glücklichste Gedanke meines Lebens"—the most fortunate thought of his life: "The gravitational field has only a relative existence in a way similar to the electric field generated by magnetolectric induction. *Because for an observer falling freely from the roof of a house there exists—at least in his immediate surroundings—no gravitational field*" (Einstein quoted in Pais 178). This requires some explanation.

According to Faraday's law of induction, a bar magnet moving uniformly relative to a conducting wire induces an electric field, which, in its turn, and according to the electric force law, produces a current in the wire. However, if we change to an inertial reference frame attached to the bar magnet, then the bar magnet is at rest, and there is no longer an induced electric field. This exemplifies the relative existence of the induced electric field Einstein refers to in the last quote: its existence depends on the choice of inertial reference frame. According to Lorentz' magnetic force law, however, the electrons in the wire still produce the current, for they are now uniformly moving with the wire in the magnetic field of the static bar magnet. Consequently, this Maxwellian story exemplifies the overall relativity or frame dependence of the electric-magnetic mix I talked about before. Indeed: in a first frame, a current in a wire is described as resulting from the action of an electric field, induced by a *changing* magnetic

field, on electrons *at rest*; in a second frame, there is *no* induced electric field, but the same current is described as resulting from the action of an *unchanging* magnetic field on uniformly *moving* electrons.

Now consider a man on the roof of a house. He experiences a gravitational force. However, when he freely falls from the roof, he does not. So, if we change to an accelerating frame attached to the freely falling man, there is no gravitational force field. The disappearance of the gravitational field in this story, when changing to the accelerated frame, seems quite similar to the disappearance of the electric field in the Maxwellian story, when changing to the uniformly moving frame. Clearly, what Einstein hit upon in 1907 is an analogy, and this analogy can be extended when taking into account inertial force fields and magnetic fields.

Consider a man enclosed in an elevator. At first, the elevator is at rest on earth, and the man feels the gravitational pull of the earth. At a later time, however, the elevator is far away from any source of gravitation, but appropriately accelerated upward, so that the man experiences a downward pull that feels exactly the same as before. The point is that in this second case the pull is described as an inertial force (a force due to inertia, to a body's resistance to acceleration, like the well-known centrifugal force), and not as a gravitational force. This elevator thought experiment also shows a similarity with the Maxwellian story. The two descriptions of pull in the elevator, one in terms of gravitational force, and the other in terms of inertial force, seems quite similar to the two descriptions of current in the wire, one in terms of an electric force, and the other in terms of a magnetic force. Consequently, the gravity-electricity analogy can be extended to hold that the accelerated frame dependence of the gravity-inertia mix is quite similar to the inertial frame dependence of the electric-magnetic mix.

One can imagine the strong impression of this analogy on Einstein. In a flash, Einstein saw that it might contain the seed to fix in one stroke the two shortcomings of the STR. The ideas that electricity and magnetism refer to a single, electromagnetic reality, and that physical laws can be inertial frame independent, had led Einstein in 1905 to a new vision of spacetime, and to a new mechanics, replacing Newton's mechanics. In 1907, the possibility dawned, that gravitation and inertia also refer to a single, inertio-gravitational reality; that physical laws can be frame independent in general; and that all this—maybe by reconsidering and changing our concepts of space and time again—might lead to a new theory of gravitation, replacing Newton's theory of gravitation.

Einstein called the intimate connection between gravitation and inertia “the equivalence principle,” and the general frame independence of physical laws “the general principle of relativity.” He hoped these principles would lead him to a general theory of relativity, including a new law of gravitation. Einstein was not immediately, and not completely, successful, even though a well-known fact of physics immediately gained intelligibility—the equality of gravitational mass (a body’s susceptibility to gravity) and inertial mass (a measure of inertia, of a body’s resistance to acceleration). This equality entails Galileo’s observation that all bodies on earth fall with the same acceleration, and hence, it also makes possible the elevator thought experiment, for suppose all bodies do not fall with the same acceleration on earth, whereas they do in the accelerated elevator, then the two situations are not really physically equivalent, but can easily be distinguished by the man in the elevator. If gravitation and inertia are indeed manifestations of one and the same inertio-gravitational reality, then the equality of inertial and gravitational mass is no coincidence, but can be understood by physicists as the consequence of a deeper unity.

It actually took Einstein from November 1907 (the most fortunate thought) to November 1915 to arrive at GTR as we know it today, and even then it was not exactly what he had hoped for. Despite its name, GTR did not extend the principle of relativity from inertial to arbitrary moving frames. This failure is connected with the fact that each analogy expresses similarity amid *diversity*, and Einstein’s 1907 analogy is no exception to that rule. Compare the conducting wire experiment and the elevator experiment again. The former case is about a unique current, described from two different reference frames (by observers who are not disabled), whereas the latter case is actually about two different pulls, arising in two different situations (which the observer in the elevator cannot distinguish because of the equality of inertial and heavy mass, but also because he is enclosed). The current is a unique particular instance of the general notion of current, described by the same theory (Maxwell’s) from two points of view. The pull on earth and the pull in outer space, however, are two different particular instances of the general notion of pull, described by the same observer by means of two different theories (Newton’s theory of gravitation, and Newtonian or special relativistic mechanics, different theories, even though connected by the equality of gravitational and inertial mass). So, whereas the conducting wire experiment is indeed about

the principle of special relativity, the elevator experiment is not about the principle of general relativity.

That it took Einstein eight years to come up with GTR is also connected with the diversity characterizing the analogy that guided his search. Again, compare the conducting wire experiment and the elevator experiment. In the wire case, the current-effect of the induced electric field in the first frame can be *globally* reproduced in the second frame, in the absence of any induced electric field. But in the elevator case, the effect of the gravitational field on earth can only be *locally* reproduced in outer space, in the absence of any gravitational field.

To explain this locality claim, I add that the man in the elevator performs one and the same experiment on earth and in outer space, namely, the experiment of dropping two balls, one from his left hand, and one from his right hand. So, first he drops the two balls when the elevator is at rest on earth, and (by means of very sophisticated instruments) he observes that the two trajectories are not perfectly parallel. Indeed (representing the earth by a perfect sphere for argument's sake) both balls fall toward the center of the earth, and the straight lines representing their trajectories intersect in that center. To put it differently, for each ball, the gravitational force acting on it can be split into a force perpendicular to the elevator floor, and a "tidal" force, horizontal to the floor, and pointing to the other ball. Secondly, the man also drops the two balls when the elevator is accelerated in outer space, and then—obviously—he observes that the trajectories of the two balls are perfectly parallel. In other words, no tidal effect is observed. Consequently, we can conclude that the gravitational field of the earth is not perfectly reproduced by the inertial field.

Of course, we can exclude the global tidal effect of the earth's gravitational field in our thought experiment by reducing the size of elevator, man, balls, etc., as well as the duration of the experiment, until they are all infinitely small, and then hold that this infinitesimal case is a case of perfect reproduction. However, the point is that we have then gone from a global to a local description—which is exactly what Einstein did, and what I wanted to highlight. The principle of equivalence is not a global, but only a local principle.

The local character of the equivalence principle has made the path of discovery of GTR long and winding, and GTR itself complex. If, by the appropriate choice of accelerated reference frame, it were possible for all

gravitational force fields to disappear globally, or to be simulated globally by inertial force fields, then Einstein would probably have come up with a much simpler theory soon after November 1907. But this is not possible, and Einstein actually needed eight years to come up with his new and complex theory of gravitation.

I am not going to describe the long and winding path of discovery of GTR, which does not only involve the initial analogy and principle I discussed, but a variety of mathematical and physical analogies and principles—too much to account for here. The character and complex interplay of the mathematical and physical strategies constituting Einstein's path to GTR has been studied in admirable detail by a team of leading Einstein scholars, and their account (see Renn)—consisting of four volumes, containing more than two thousand pages—cannot be summarized in a few paragraphs. The only point I wanted to make here (e.g., by presenting the equivalence principle as a conceptual principle that can be formulated in terms of Newton's theories) is that the main motives that led to the discovery of STR and GTR were conceptual, not empirical, and also, that they were rooted in non-fundamental theories—contrary to the spirit of what Curiel claims (that the discovery of quantum gravity is not likely to be rooted in a less fundamental theory that is empirically successful). As a bonus, I have been able to illustrate that in the context of discovery, the analogical nature of our conceptual thinking is even more obvious and relevant than in the context of justification.

#### *Whitehead's practice of discovery*

The importance of analogical thinking in the context of discovery, and the untenability of Curiel's conjectures with regard to the most likely path of discovery of quantum gravity, can be illustrated even better when turning our attention to Whitehead. To start with, as in Einstein's case, an analogy motivated and guided Whitehead's discovery of his alternative theory of gravitation. Secondly, in Whitehead's philosophy the role of induction as a method of discovery is explicitly revalued by changing the Baconian concept of induction into an analogy-driven concept of induction. And thirdly, Whitehead's alternative theory of gravity is a Minkowski background dependent theory, as are all quantum field theories, hence suggesting a direction for future quantum gravity research. Let me try to give a very brief account of the first two points in this section, and of the third point in the next one.

The analogy that guided Whitehead's discovery of an alternative theory of gravitation was actually an analogy made by Minkowski in his famous lecture, "Space and Time," given in 1908 in Cologne. Whitehead most likely learned about STR in 1912, and he learned about the GTR in 1916. The real impact of Minkowski's 1908 lecture on his thinking, however, can be situated, approximately, in 1918. Whitehead himself wrote to Victor Lowe, his biographer: "Minkowski's paper was published in 1908, but its influence on me was postponed approximately ten years" (Lowe 15). By 1918, Whitehead's philosophy of science was in full development, and was clearly diverging from Einstein's interpretation of STR and GTR. My conjecture is that by 1918 Whitehead was very susceptible to Minkowski's "Space and Time" lecture, and the analogy I will highlight not only fits Whitehead's own developing philosophical views, but also showed him the way to finding an alternative rendering of GTR.

It was in his "Space and Time" lecture that Minkowski said: "Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality" (75). In "Space and Time," Minkowski expressed his vision of the whole universe as being resolved into world-lines in spacetime, and of a world-line as "the everlasting career of the substantial point" (76). Moreover, in "Space and Time" Minkowski held that "physical laws might find their most perfect expression as reciprocal relations between those world-lines" (76) and after describing the electro-dynamical relations between the world-lines of point-charges in terms of the Maxwell-Lorentz electron theory and the Liénard-Wiechert retarded potentials, Minkowski expressed his belief that the resolution of the universe in world-lines of point-charges can be seen as "the true nucleus of an electromagnetic image of the world" (91). Finally, but most importantly, in "Space and Time" Minkowski also conjectured that the gravitational relations between the world-lines of point-masses should be treated just like the electromagnetic relations in the case of point-charges, and he accordingly proposed a relativistic law of gravitation.

The law of gravitation Minkowski proposed in 1908 did not survive. For one thing, it failed to explain the observed precession of the perihelion of Mercury. But his analogy, relating the gravitational relations between world-lines of point-masses with the electromagnetic relations between world-lines of point-charges, made quite an impression on Whitehead. Like Einstein's discovery analogy, Minkowski's analogy links the semantic

target domain of gravitation with the semantic source domain of electromagnetism. Einstein's analogy implies a local equivalence of gravitation and inertia, which ultimately entails the merger of gravitational physics and spacetime geometry in a fundamental tensor describing the variable metric and curvature of a kind of inertio-gravitational substance, interacting with material substances and electromagnetic fields. However, the Minkowski-Whitehead analogy is quite different. It implies that gravitational relations, like electromagnetic relations, can be conceived as relations of world-lines—or in Whitehead's terminology: historical routes—in the uniform spacetime of STR. According to Whitehead, this way of conceiving things, contrary to Einstein's, allows GTR to satisfy not only the empirical criterion of experimental success, but also the intelligibility criterion of experiential coherence, because it dovetails with an ontology of actual occasions, conceived in coherence with the occasions constituting each stream of human experience.

Let us now focus on the question of how an analogy can be the motor of a discovery. In general, an analogy offers the possibility to better understand a particular situation by its similarity to another particular situation. Consider again Einstein's analogy. Einstein was struck by the similarity between the fact that a man is not subject to gravitational pull while he is falling from a roof, and the fact that an electron in a conducting wire is not subject to electric pull while the wire is uniformly moving toward a magnet bar at rest. The better understanding arises from the fact that the second situation belongs to a more familiar domain, a domain of which the order is more familiar. Einstein was familiar with the disappearance of induced electric fields by a change of inertial reference frame in the domain of electromagnetic phenomena because he was familiar with the order of that domain in as far as it had been captured in the Maxwell-Lorentz laws. Moreover, in the context of discovery the analogy is extended from the two particular cases to the two domains to which they belong. If the particular situation of the falling man is similar to the particular situation of the uniformly moving electron, then the order of the target domain of gravitation to which the falling man belongs might be similar to the order of the source domain of electromagnetism to which the electron belongs. Hence, the theory of gravitation might be similar to the theory of electromagnetism. And while exploring that similarity, new particular cases are put to the fore. If, for example, we can make a gravitational force disappear by accelerating the reference frame as if it was an electric

force disappearing by uniformly moving the reference frame, and if a similar order characterizes the domain of gravitation and the domain of electricity, then there probably exists a force replacing the gravitational force in the role of pulling a man in an appropriately accelerating elevator because, in the electromagnetic environment, the magnetic force replaces the electric force in the role of generating a current in an appropriately moving wire.

According to Whitehead's account of how analogies fuel the motor of discovery, similarity of present events (actual occasions) might imply similarity of the (spatio-temporal and contingently) ordered environments (societies) to which they belong, and the understanding thus gained about the environment under investigation can lead to prediction of the character of future events belonging to that environment. The reason why similarity of events might imply similarity of their environments is not different from the reason why the prediction of the character of a future event within the environment under investigation might be successful. According to Whitehead, the reason is that events—especially the events on which physicists focus—largely inherit their character from being part of an ordered environment. In other words, the reason is that events largely acquire their character by the causal efficacy of past events belonging to their environment. This immediately implies Whitehead's critique of both Bacon's notion of induction and Hume's analysis of causality.

Baconian induction is about observing many events of the same type, in the hope of discovering a pattern characterizing all events of that type, past and future. It is about repeated observation of particular events of one and the same environment with the aim of deducing a general law applicable to both past and future of that environment. However, Baconian induction cannot cope with the facts that all observation in physics is theory-laden (so: what existing theory allows for observation prior to discovering a new theory?), that the induced theory is not merely formulated in observational terms (so: what explains the jump from observational to theoretical terms?), and that a pattern that occurred in the past may well fail to reoccur in the future (so: what justifies the belief in the reoccurrence of past patterns?). Of course, the last issue mentioned has acquired its full weight by Hume's analysis of causation. If an instance of causation is nothing more than our observation of a particular succession of externally related events, and does not express the internal constitution of present events by past events, then it fails to justify induction. No

wonder that Whitehead wrote that “the Baconian method of induction” is “a method which, if consistently pursued, would have left science where it found it” (*PR* 5), and that he called it—especially in the light of Humean skepticism—“the despair of philosophy” (*SMW* 23).

So it will not come as a surprise that Whitehead did not simply rehabilitate Baconian induction. He wrote:

Induction has proved to be a somewhat more complex process than Bacon anticipated. He had in his mind the belief that with a sufficient care in the collection of instances the general law would stand out by itself. We know now . . . that this is a very inadequate account of the processes which issue in scientific generalizations. (*SMW* 43)

However, Whitehead did not accept Hume’s criticism, which he judged to be the result of a superficial analysis of human sense-perception. Whitehead did not limit perception to the spatio-temporal presentation of sense data, a mode of perception that he called “sense-presentation” or “presentational immediacy.” He also included the process from “sense-awareness” to “sense-presentation,” the process from “causal efficacy” to “presentational immediacy.” Moreover, Whitehead analyzed the process constituting each actual occasion in coherence with his analysis of the process constituting each occasion of sense-perception. Hence, Whitehead rehabilitated the notion of causality, which scientists had never abandoned anyway, despite Hume’s philosophical skepticism (and in accord with his practical advice).

Without further account of Whitehead’s divergence from Hume, I conclude that, contrary to Hume, Whitehead did “not wish to throw any doubt upon the validity of induction, when it has been properly guarded.” Whitehead wished “to justify induction,” but he added: “You will observe that I do not hold Induction to be in its essence the derivation of general laws. It is the divination of some characteristics of a particular future from the known characteristics of a particular past” (*SMW* 44—also quoted by Whitehead in *PR* 204). To understand what he meant by the latter remark is to understand that he replaced Bacon’s method of induction with his analogy-driven method of induction, which I described in simple terms a few paragraphs ago, and which is described by Whitehead (in less simple terms) in *Process and Reality* (see *PR* 199-207). Here are some revealing quotes (also quoted in the inspiring chapter on induction in Plamondon—a chapter involving a comparison of the Whiteheadian account with Mary Hesse’s account of valid inductive inference):

Inductive reasoning gains its validity by reason of a suppressed premise. This tacit presupposition is that the particular future which is the logical subject of the judgment, inductively justified, shall include actualities which have close analogy to some contemporary subject enjoying assigned experience. . . . It is also presumed that this future is derived from the present by a continuity of inheritance in which this condition is maintained. There is thus the presupposition of the maintenance of the general social environment. (*PR* 204)

An inductive argument always includes a hypothesis, namely, that the environment which is the subject-matter considered contains a society of actual occasions analogous to a society in the present. . . . Thus the laws of nature dominating the environment in question have some analogy to the laws of nature dominating the immediate environment. (*PR* 205)

Thus the basis of all probability and induction is the fact of analogy between an environment presupposed and an environment directly experienced. (*PR* 207)

Whitehead's notion of induction by analogy and causality provides an answer to each of the questions that discredit Bacon's account of induction. To start with: what existing theory allows for observation prior to the discovery of a new theory? It is the familiar theory applying to the particular event that is analogous to the one we wish to improve our understanding of, or in other words, analogous to the one a new theory is being searched for. Secondly: what explains the jump from observational to theoretical terms? There is no such jump. What happens is that the terms of the familiar theory are transformed into the terms of the new one. Familiar theoretical terms are stretched in order to apply to an environment, and often even a level of generality, "foreign to their ordinary usage." And Whitehead adds: "however such elements of language be stabilized as technicalities, they remain metaphors mutely appealing for an imaginative leap" (*PR* 4). For Whitehead the major shortcoming of Bacon's method of induction is indeed the omission of the analogical or metaphorical imagination, and he writes: "what Bacon omitted was the play of a free imagination, controlled by the requirements of coherence and logic" (*PR* 5). And finally: what justifies the belief in the reoccurrence of past patterns? It is the fact that patterns of order can be maintained by a continuity of inheritance, or in other words, by a dominance of causal efficacy.

*Feynman's practice of discovering quantum gravity*

One of the aspects of GTR, an aspect Whitehead always considered of major importance, is that it necessitates a change in the theory of electromagnetism. Whitehead wrote: "it is an outcome of Einstein's work that the electro-magnetic equations require modification to express the association of the gravitational and electro-magnetic fields. This is one of his greatest discoveries. The most natural deduction to make from these modified equations is that the velocity of light is modified by the gravitational properties of the field through which it passes" (*ESP* 334). Or again: "the electromagnetic theory has to be modified to allow for the presence of a gravitational field. Thus Einstein's investigations lead to the first discovery of any relation between gravity and other physical phenomena" (*CN* 183). Consequently: "in the absence of gravitational fields, Whitehead adopts the standard Maxwell-Lorentz field equations for electromagnetic phenomena. However, in the presence of a gravitational field, Whitehead modifies the Maxwell-Lorentz equations to take account of the influence of gravitation on electromagnetic phenomena" (Palter 204). Moreover, the awareness of the interrelatedness of gravitational and electromagnetic fields of force prompted Whitehead to aim at a unified field-treatment of gravitation and electromagnetism, as it prompted Einstein to aim at a geometrical unification of gravitation and electromagnetism.

My above account of the analogy-driven nature of discovery has insufficiently stressed that analogies work both ways. Einstein's new theory of gravitation was not only shaped by Einstein's guiding analogy with electromagnetism. It also pushed Einstein to reshape electromagnetism. A fertile analogy appeals for an interaction between its semantic fields, between source and target domain, which can lead to an imaginative leap in both of them. Einstein's theory of gravitation, GTR, was not simply a copy of the Maxwell-Lorentz theory of electromagnetism. It challenged physicists to have new look at electromagnetism, and hence, to revise the theory that had acted as the initial model for GTR. So, even apart from the mutual influence of gravitation and electromagnetism, Einstein's followers—Hermann Weyl to start with—were inspired to come up with a geometrical description of electromagnetic phenomena, leading to a unification of gravitation and electromagnetism. However, no successful geometrical unification was arrived at. Moreover, with the rise of quantum mechanics, and with the discovery of strong and weak nuclear forces, things got a lot more complicated. Here is what Richard Feynman

said about the situation when he gave his *Lectures on Gravitation* in the academic year 1962-63:

Einstein's gravitational theory, which is said to be the greatest single achievement of theoretical physics, resulted in beautiful relations connecting gravitational phenomena with the geometry of space; this was an exciting idea. The apparent similarity of gravitational forces and electric forces, for example, in that they both follow inverse-square laws, which every kid can understand, made every one of these "kids" dream that when he grew up, he would find the way of geometrizing electrodynamics. Thus a generation of physicists worked trying to make a so-called unified field theory, which would have unified gravitation and electrodynamics into a single thing. None of these unified field theories has been successful. . . . Einstein himself worked at this, . . . but nevertheless, there is no successful unified field theory that combines gravitation and electrodynamics. Such a success would have been short lived, however, because now we have so much more in physics than just electrodynamics and gravitation. (2)

Feynman himself decided to take an alternative approach. Instead of trying to geometrize quantum electrodynamics (which meanwhile had replaced the Maxwell-Lorentz electrodynamics in the domain of quantum physics) because GTR, and its geometrical interpretation, were so successful and stable, he tried to quantize GTR (an approach Curiel discredits). So instead of taking the geometrical approach of GTR as the model to revise quantum electrodynamics, he took the quantum field approach of quantum electrodynamics as the model to try to transform GTR into quantum gravity. His *Lectures on Gravitation* illustrate Feynman's alternative approach, and Brian Hatfield stresses the original character of Feynman's starting point:

The standard and historical approach to classical gravitation is to start with the Principle of Equivalence and to develop the geometrical viewpoint. Feynman was proud of the fact that he seldom followed the standard approach. On the corner of one blackboard in his office he wrote: "What I cannot create, I do not understand." . . . Thus it is no surprise that Feynman would recreate general relativity from a nongeometrical viewpoint. . . . When the ultimate goal is to quantize gravity, Feynman felt that the geometrical interpretation just stood in the way. (xxxii)

For someone familiar with Whitehead's writings on GTR, Feynman's decision to "take a different approach to the subject" (Feynman 1),

instead of starting with the equivalence principle, sounds like an echo of Whitehead's refusal to start from "the clue by which Einstein guided himself along the path [of discovery]" (*ESP* 332). The equivalence of inertia and gravity ultimately led Einstein to the identification of the spacetime manifold and the gravitational field. Hence, dropping that equivalence as the starting point makes possible a non-geometrical field approach, and Feynman's approach, like Whitehead's, is indeed such a non-geometrical approach.

According to Feynman: "The geometric interpretation is not really necessary or essential to physics" (113). However, unlike Whitehead, Feynman does not want "to maintain the old division between geometry and physics" (*R v*). On the contrary, once his alternative derivation of Einstein's formulae is completed, "geometrical ideas enter Feynman's discussion . . . through 'the back door'" (Preskill and Thorne xi). In other words, whereas Whitehead does not go beyond his action (or impetus) interpretation of the tensor formalism in order to embrace Einstein's geometrical interpretation, Feynman does:

So, for example, the Riemannian curvature tensor (a centerpiece of the conventional formulation of general relativity) is introduced by Feynman initially only as a device for constructing terms in the gravitational action with desired invariance properties. Not until [later in his *Lectures on Gravitation*] does Feynman reveal that the curvature has an interpretation in terms of the parallel transport of a tangent vector on a curved spacetime manifold. (Preskill and Thorne xi)

In Charles Misner, Kip Thorne, and John Wheeler's "bible" of gravitation, Feynman's approach to gravitation is correctly listed as one of the "attempts to incorporate gravity into special relativity," but aware of the fact that it ultimately fused with Einstein's geometrical approach, these authors hasten to add that by his "field-theory route," Feynman arrived "at standard 1915 general relativity," and "that what ostensibly started out as a flat-space theory of gravity is really Einstein's theory, with gravitation being a manifestation of the curvature of spacetime" (177-78). Misner, Thorne, and Wheeler—the last of these being "the leading developer of contemporary geometrodynamics" (Graves 10)—are happy to recognize "Einstein's line of reasoning" (Misner et al. 178) at the end point of Feynman's line of reasoning, whereas I prefer to focus on the Whiteheadian character of its starting point.

Feynman's approach, like Einstein's approach and Whitehead's approach, is based on a gravitation-electrodynamics analogy. As will be clear from what has already been said, this analogy is not the one underlying Einstein's equivalence principle. Feynman's analogy is closer to Whitehead's, even though, of course, it is not simply the analogy Minkowski launched in "Space and Time," linking gravitational relations to classical electrodynamic relations of interaction between world-lines. The retarded potential interaction between point-charges is replaced by the exchange of force-mediating particles between those charges—in other words, classical electrodynamics is replaced by quantum electrodynamics. In their "Foreword" to Feynman's *Lectures on Gravitation*, John Preskill and Kip Thorne write:

At the time of these lectures, Feynman was struggling to quantize gravity—that is, to forge a synthesis of general relativity and the fundamental principles of quantum mechanics. Feynman's whole approach to general relativity is shaped by his desire to arrive at a quantum theory of gravitation as straightforwardly as possible. For this purpose, geometrical subtleties seem a distraction; in particular, the conventional geometrical approach to gravitation obscures the telling analogy between gravitation and electrodynamics.

With hindsight, we can arrive at Maxwell's classical electrodynamics by starting with the observation that the photon is a massless spin-1 particle. The form of the quantum theory of a massless spin-1 particle coupled to charged matter is highly constrained by fundamental principles such as Lorentz invariance and conservation of probability. The self-consistent version of the quantum theory—quantum electrodynamics—is governed, in the classical limit, by Maxwell's classical field equations. Emboldened by this analogy, Feynman views the quantum theory of gravitation as 'just another quantum field theory' like quantum electrodynamics. Thus he asks . . . : can we find a sensible quantum field theory describing massless spin-2 quanta (gravitons) coupled to matter, in ordinary flat Minkowski spacetime? The classical limit of such a quantum theory should be governed by Einstein's general relativistic field equation for the classical gravitational field. (x-xi)

It is possible to define the Lorentz invariant field of massless spin-1 particles (photons) coupled to charged matter to arrive at quantum electrodynamics, which—in the classical limit—amounts to Maxwell's equations. By taking his distance from Einstein's equivalence principle, and from the resulting geometrical interpretation of GTR, Feynman was

able to approach gravitation by analogy with the quantum electrodynamics approach just sketched. He defined the Lorentz invariant field of massless spin-2 particles (gravitons) coupled to the energy-momentum tensor of matter to arrive at a quantum field theory of gravitation, yielding—in the classical limit—Einstein's equation of gravitation. In other words, by following (in complete independence) a Whiteheadian approach, he arrived at a quantum gravity proposal, which reduces to Einstein's equations in the classical limit.

It is important to add that, like Whitehead's theory of gravitation, Feynman's theory of quantum gravity is a background dependent theory. In its initial, non-geometrical interpretation, like all quantum fields, Feynman's quantum gravity field is defined against the background of Minkowski's spacetime geometry. Moreover, neither Whitehead's theory, nor Feynman's theory, has withstood the test of time, and the alternative search for a background independent theory of quantum gravity is more popular nowadays. However, given the overall lack of success so far to formulate a generally acceptable theory of quantum gravity, and given Whitehead's philosophical critique of Einstein's background independent interpretation of GTR, the Whitehead-Feynman approach can still be taken as a guiding example for future research. Consequently, I have given ample reason to cast doubt upon Curriel's view that the search for quantum gravity is not furthered by the search for an alternative interpretation of GTR, driven by analogical reasoning instead of adverse experimental data.

#### *The measurement problem of GTR*

According to Curriel, quantum mechanics is in need of an interpretation because of the measurement problem, because of the physicist's inability to give a genuine quantum theoretical description of the measurement process. The so-called decoherence approach may be the most promising of the many responses to the need for an appropriate interpretation of quantum mechanics. However, no consensus in the physics community has been reached on the subject, and this lack of consensus implies that it would be wrong to conclude that the measurement problem of quantum mechanics has been solved. Hence, the search for an appropriate interpretation of quantum mechanics has to be continued.

Curriel also holds that no similar problem exists for GTR, and, consequently, that the search for an appropriate interpretation of GTR cannot

be motivated by appeal to a general relativistic measurement problem. However, at this point, as I already highlighted by means of a couple of quotes, Einstein would disagree with Curiel. Einstein was aware that he introduced measuring-rods and clocks in GTR as if they are self-sufficient elements, whereas they should be introduced as theoretical constructions (e.g., configurations of field-interacting particles). Hence, he saw it as an obligation to eliminate at a later stage what he called the inconsistent introduction of measurement entities, such as rods and clocks, which are independent of the fundamental theoretical entities, such as particles and fields.

Like Einstein, Whitehead would disagree with Curiel. For Whitehead, the introduction of self-sufficient entities is the mark of incoherence (and the acceptance of such incoherence, the mark of dogmatism). However, his requirement of coherence in this context is not Einstein's request to define the measurement *entities* of the theory in terms of its fundamental physical entities.

According to Whitehead, each of our direct measurement acts involves a sense-perceptual judgment of spatio-temporal congruence, belonging to a larger class of direct judgments or recognitions of "constancy" and "matching" (*PNK* 54-57), "sameness" and "identity" (*CN* 124-26), "equality" (*R* 40-41) and "permanence" (*PR* 327-29), and I would add, "analogy," for Whitehead writes that "equality embraces in its scope . . . identity and diversity," and he adds that "the important use of equality is when there is diversity of things related and identity of character" (*R* 41-42). Hence, a measurement theory is coherent with the general character of our sense perception in the mode of presentational immediacy if its spacetime geometry coheres with the spatio-temporal congruence patterns recognized in our direct measurement **acts**.

Whitehead, in *An Enquiry Concerning the Principles of Natural Knowledge* (1919) and *The Concept of Nature* (1920), listed the requirements for spacetime geometry to cohere with those patterns as geometrical axioms, and he showed that Minkowski's constantly curved spacetime geometry of zero curvature satisfies these axioms. At a later stage (*The Principle of Relativity*, 1922), however, he seems to have evolved to thinking that constantly curved spacetime geometries of non-zero curvature might also cohere with those patterns, for he explicitly states that his coherence requirement boils down to the requirement for spacetime to be uniform, and still leaves room for a conventional choice with regard to the type of

constant curvature (see *Rv*). Anyway, the variable curvature of Einstein's spacetime immediately reveals that Einstein's interpretation of GTR is not coherent with the general character of our direct experience, because it is not uniform. To meet Whitehead's coherence requirement (as expressed in *The Principle of Relativity*), a reinterpretation of GTR in terms of a constantly curved spacetime is wanted. Of course, Whitehead's own interpretation of GTR in terms of Minkowski's zero curved spacetime is such a reinterpretation. But George Temple's 1923 interpretations of GTR in terms of non-zero curved spacetimes also fit the bill ("Generalization").

It is interesting to note that, like Einstein and Whitehead, Weyl was vividly aware of the measurement problem of GTR (e.g., it is the central issue in Weyl 1949). However, in order to solve it, Weyl replaced Einstein's ordinary measurement rods and clocks by infinitesimal physical processes, and he replaced the ordinary notion of congruence (to which Whitehead refers) with the notion of infinitesimal congruence (see Temple, *100 Years* 89-90, and Ryckman 85-86). Weyl's *reine Infinitesimalgeometrie* is a generalization of pseudo-Riemannian geometry, and it led to the first geometrical attempt to unify gravitation and electromagnetism. Even though Einstein initially judged Weyl's geometrical interpretation of GTR to be "a stroke of genius of the highest magnitude," he rejected Weyl's solution of the measurement problem because of its alleged empirical inconsistency with the constancy of atomic spectral lines (see Ryckman 80-81 and 86-89). And even though Weyl, like Whitehead, defined measurement in terms of congruence, Whitehead rejected Weyl's solution of infinitesimal or *local* congruence because of its rational incoherence with the *global* congruence patterns recognized in our direct measurement acts.

Each of Whitehead's London books—all three dealing with his philosophy of science in general, and his interpretation of relativity in particular—contains a chapter on the topic of global congruence-based measurement (see *PNK* Chapter IV, *CN* Chapter VI, *R* Chapter III), and so does *Process and Reality* (Part IV, Chapter V). Even though Whitehead never mentions Weyl by name in his writings, he was familiar with Weyl's work (at least indirectly, e.g., via Haldane), and I am quite sure he addressed Weyl when he wrote: "The modern answer [to the problem of measurement] is that measurement is a comparison of infinitesimals. . . . The answer to this answer is that there are no infinitesimals . . ." (*PR* 328). No direct experience (sense perception in the mode of presentational immediacy) of infinitesimals is possible. Moreover, Weyl's pure

mathematical intelligibility criterion of infinitesimal congruence (which dovetails with Weyl's transcendental-phenomenological idealism, see Ryckman 2005:108-44) allows for variable curvature, whereas Whitehead's experiential intelligibility criterion of global congruence only allows for constant curvature.

Anyway, whether we focus on Whitehead, or on Einstein or Weyl, one thing is clear: GTR is not without a measurement problem. In *Whitehead and the Measurement Problem of Cosmology*, Gary Herstein calls it "a fundamental problem at the heart of general relativity," which does not involve "the empirical adequacy per se of general relativity," but its incoherence. (9). The fact that Einstein's interpretation of GTR does not cohere with the general character of our direct measurement acts implies that it is at variance with the general character of the practices that are supposed to supply its evidence. In this sense, Einstein's interpretation of GTR, as Herstein does not fail to stress, "cannot legitimize the results it produces." (10)

Again, GTR is not without a measurement problem, and in this respect, GTR is as much in need of an appropriate interpretation as is quantum mechanics. This means that the last of Curiel's list of arguments (and hence, by now, his whole list) to dispose of interpretation efforts in the context of GTR has been rendered highly doubtful.

## CONCLUSION

The aim of this essay was to defend the research of Whitehead's alternative interpretation of GTR against the claim that GTR is not in need of an interpretation. I have shown that the restriction of GTR to its pure and uninterpreted mathematical kernel falls pray to the myth of a purely abstract understanding of nature, and that the restriction of the set of interpretations of GTR to the one and only Einsteinian interpretation is haunted by a number of serious issues. Einstein's initial interpretation, for example, strengthens the bifurcation of nature into the world of geometrical physics and the world of human experience, it gives rise to the circularity that each instance of solving Einstein's field equation presupposes its actual solution, and it is unable to solve the measurement problem of GTR, which is not unlike the one haunting quantum mechanics.

The claim to entertain no, or only one, interpretation draws GTR outside the sphere of human creativity, and it reduces the theory to a vehicle for mere experimental application. This reductive claim shields GTR from

conceptual discussions, analogical enrichment, and metaphorical discoveries, hence shielding physics from educational and scientific progress.

Einstein never intended to be led into such dogmatic shielding, and he was aware or became aware that both the vicious circle and the measurement issue are in need of a solution. Feynman was no dogmatist either, but rather the exact opposite, and his educational lectures on gravitation highlighted the possibility to recover Einstein's equations by means of a quantum field approach to gravitation, which is Minkowski background dependent. Both Einstein's awareness of some of the shortcomings of his own theory and Feynman's background dependent field-approach are in accord with some important (even though not all) aspects of Whitehead's critique of Einstein's initial interpretation, and of his approach to arrive at an alternative interpretation of GTR. Thus, Whitehead's writings on the subject deserve renewed efforts in scholarship and interpretation.

#### ENDNOTES

1. Curiel's opinion that GTR is not confronted with adverse experimental data can be challenged as well. One might argue that the presence or absence of adverse experimental data is linked to the choice of experimental protocol, and that in order to obtain adverse experimental data, gravity experiments are needed whose experimental protocol does not presume the geometric interpretation.

2. Again, if experimental protocols presuppose the paradigm interpretation, it is hard to obtain empirical data that show us where it goes wrong.

3. Of course, Valéry was no mathematician, but his aphorism beautifully captures the essence of mathematics as expressed in Eugene Wigner's remark "that mathematics is the science of skillful operations with concepts and rules invented just for this purpose" (2).

4. This 1925 remark of Whitehead from *Science and the Modern World* is an echo of Whitehead's 1911 remark in *Introduction to Mathematics*, that pure geometry is not about "the 'spaciness' of space," and that its properties "do not involve any peculiar space-apprehension or space-intuition or space-sensation" (*IM* 180).

5. For example, taking the set of all triplets of real numbers, both the function mapping  $(x,y,z)$  and  $(u,v,w)$  to a positive real number by means of the Pythagorean formula

$$\sqrt{(x-u)^2 + (y-v)^2 + (z-w)^2}$$

and the function thus mapping them by means of

$$\max \left\{ \sqrt{(x-u)^2}, \sqrt{(y-v)^2}, \sqrt{(z-w)^2} \right\}$$

are distance functions, for they both meet the three relevant conditions. Now, geometrically represent a sphere (say, the sphere with center (0,0,0) and radius one) by means of a Cartesian coordinate frame, first according to the Pythagorean distance function, and then according to the maximum distance function. Then you will first see a spherical shape, and then a cubical one. The latter shape circumscribes the former (see Van Bendegem 140).

6. For an example of a contemporary Pythagorean view, however, see Tegmark.

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